1. Let $X$ be the real line with the particular point topology for the point 0, and let $Y$ be the real line with the particular point topology for the point 1. What is the product topology on $X \times Y$?

2. Consider the diagonal map $\Delta : X \to X \times X$, $\Delta(x) = (x,x)$, where $X$ is some topological space and $X \times X$ has the product topology. a) Prove that $\Delta$ is continuous. b) Prove that $X$ is Hausdorff if and only if $\Delta(X)$ is closed in $X \times X$.

3. We know that the projection maps send open sets to open sets. Do they send closed sets to closed sets?

4. Prove that $X \times Y$ is Hausdorff if and only if both $X$ and $Y$ are Hausdorff.