

Tutorial Sheet 5, Topology 2013

1. Let X be the real line with the particular point topology for the point 0, and let Y be the real line with the particular point topology for the point 1. What is the product topology on $X \times Y$?
2. Consider the diagonal map $\Delta : X \rightarrow X \times X$, $\Delta(x) = (x, x)$, where X is some topological space and $X \times X$ has the product topology. a) Prove that Δ is continuous. b) Prove that X is Hausdorff if and only if $\Delta(X)$ is closed in $X \times X$.
3. We know that the projection maps send open sets to open sets. Do they send closed sets to closed sets?
4. Prove that $X \times Y$ is Hausdorff if and only if both X and Y are Hausdorff.