Tutorial Sheet 6, Topology 2013

1. Prove that a space X is connected if and only if there do not exist nonempty disjoint sets A and B (not necessarily open) such that $\bar{A} \cap B = A \cap \bar{B} = \emptyset$ and whose union is X.

2. Are the following sets connected?

   (a) The rational numbers, considered as a subset of the real numbers.

   (b) The subset of $\mathbb{R}^2$ defined by

   $$X = \{(x, y) : y = 0\} \cup \{(x, y) : x > 0 \text{ and } y = 1/x\}$$

   (c) Any set with the discrete topology.

3. Let $f : [a, b] \to \mathbb{R}$ be continuous with $f(a) < 0 < f(b)$. Use the connectedness of $[a, b]$ to prove the intermediate value theorem: there must be a $c \in (a, b)$ such that $f(c) = 0$.

4. Prove that the continuous image of a connected set is connected.