Homework Assignment 5, Due Thursday, Dec 3

1) (From Evans) Suppose U is connected and $u \in W^{1,p}(U)$ satisfies $Du = 0$ almost everywhere. Prove that u is constant almost everywhere in U.

2) (From Evans) Use the Fourier transform to prove that, if $u \in H^s(\mathbb{R}^n)$ for $s > n/2$, then $u \in L^\infty(\mathbb{R}^n)$ with the bound
$$\|u\|_{L^\infty(\mathbb{R}^n)} \leq C\|u\|_{H^s(\mathbb{R}^n)},$$
where $C = C(s, n)$.

3) Note that the space $W^{1,2}(\mathbb{R}^2)$ is borderline for the Sobolev embedding theorem, in the sense that $k = n/p$. Prove that $W^{1,2}(\mathbb{R}^2)$ is not a subset of $C(\mathbb{R}^2)$ by constructing an element of $W^{1,2}(\mathbb{R}^2)$ that is unbounded at the origin.