

## Exercises 13.2

### Integrating Vector-Valued Functions

Evaluate the integrals in Exercises 1–10.

- $\int_0^1 [t^3\mathbf{i} + 7\mathbf{j} + (t + 1)\mathbf{k}] dt$
- $\int_1^2 \left[ (6 - 6t)\mathbf{i} + 3\sqrt{t}\mathbf{j} + \left(\frac{4}{t^2}\right)\mathbf{k} \right] dt$
- $\int_{-\pi/4}^{\pi/4} [(\sin t)\mathbf{i} + (1 + \cos t)\mathbf{j} + (\sec^2 t)\mathbf{k}] dt$
- $\int_0^{\pi/3} [(\sec t \tan t)\mathbf{i} + (\tan t)\mathbf{j} + (2 \sin t \cos t)\mathbf{k}] dt$
- $\int_1^4 \left[ \frac{1}{t}\mathbf{i} + \frac{1}{5-t}\mathbf{j} + \frac{1}{2t}\mathbf{k} \right] dt$
- $\int_0^1 \left[ \frac{2}{\sqrt{1-t^2}}\mathbf{i} + \frac{\sqrt{3}}{1+t^2}\mathbf{k} \right] dt$
- $\int_0^1 [te^{t^2}\mathbf{i} + e^{-t}\mathbf{j} + \mathbf{k}] dt$
- $\int_1^{\ln 3} [te^t\mathbf{i} + e^t\mathbf{j} + \ln t\mathbf{k}] dt$
- $\int_0^{\pi/2} [\cos t\mathbf{i} - \sin 2t\mathbf{j} + \sin^2 t\mathbf{k}] dt$
- $\int_0^{\pi/4} [\sec t\mathbf{i} + \tan^2 t\mathbf{j} - t \sin t\mathbf{k}] dt$

### Initial Value Problems

Solve the initial value problems in Exercises 11–16 for  $\mathbf{r}$  as a vector function of  $t$ .

- Differential equation:  $\frac{d\mathbf{r}}{dt} = -t\mathbf{i} - t\mathbf{j} - t\mathbf{k}$   
Initial condition:  $\mathbf{r}(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
- Differential equation:  $\frac{d\mathbf{r}}{dt} = (180t)\mathbf{i} + (180t - 16t^2)\mathbf{j}$   
Initial condition:  $\mathbf{r}(0) = 100\mathbf{j}$
- Differential equation:  $\frac{d\mathbf{r}}{dt} = \frac{3}{2}(t+1)^{1/2}\mathbf{i} + e^{-t}\mathbf{j} + \frac{1}{t+1}\mathbf{k}$   
Initial condition:  $\mathbf{r}(0) = \mathbf{k}$
- Differential equation:  $\frac{d\mathbf{r}}{dt} = (t^3 + 4t)\mathbf{i} + t\mathbf{j} + 2t^2\mathbf{k}$   
Initial condition:  $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$
- Differential equation:  $\frac{d^2\mathbf{r}}{dt^2} = -32\mathbf{k}$   
Initial conditions:  $\mathbf{r}(0) = 100\mathbf{k}$  and  $\left. \frac{d\mathbf{r}}{dt} \right|_{t=0} = 8\mathbf{i} + 8\mathbf{j}$

- Differential equation:  $\frac{d^2\mathbf{r}}{dt^2} = -(\mathbf{i} + \mathbf{j} + \mathbf{k})$   
Initial conditions:  $\mathbf{r}(0) = 10\mathbf{i} + 10\mathbf{j} + 10\mathbf{k}$  and  $\left. \frac{d\mathbf{r}}{dt} \right|_{t=0} = \mathbf{0}$

### Motion Along a Straight Line

- At time  $t = 0$ , a particle is located at the point  $(1, 2, 3)$ . It travels in a straight line to the point  $(4, 1, 4)$ , has speed 2 at  $(1, 2, 3)$  and constant acceleration  $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ . Find an equation for the position vector  $\mathbf{r}(t)$  of the particle at time  $t$ .
- A particle traveling in a straight line is located at the point  $(1, -1, 2)$  and has speed 2 at time  $t = 0$ . The particle moves toward the point  $(3, 0, 3)$  with constant acceleration  $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ . Find its position vector  $\mathbf{r}(t)$  at time  $t$ .

### Projectile Motion

Projectile flights in the following exercises are to be treated as ideal unless stated otherwise. All launch angles are assumed to be measured from the horizontal. All projectiles are assumed to be launched from the origin over a horizontal surface unless stated otherwise.

- Travel time** A projectile is fired at a speed of 840 m/sec at an angle of  $60^\circ$ . How long will it take to get 21 km downrange?
- Range and height versus speed**
  - Show that doubling a projectile's initial speed at a given launch angle multiplies its range by 4.
  - By about what percentage should you increase the initial speed to double the height and range?
- Flight time and height** A projectile is fired with an initial speed of 500 m/sec at an angle of elevation of  $45^\circ$ .
  - When and how far away will the projectile strike?
  - How high overhead will the projectile be when it is 5 km downrange?
  - What is the greatest height reached by the projectile?
- Throwing a baseball** A baseball is thrown from the stands 32 ft above the field at an angle of  $30^\circ$  up from the horizontal. When and how far away will the ball strike the ground if its initial speed is 32 ft/sec?
- Firing golf balls** A spring gun at ground level fires a golf ball at an angle of  $45^\circ$ . The ball lands 10 m away.
  - What was the ball's initial speed?
  - For the same initial speed, find the two firing angles that make the range 6 m.
- Beaming electrons** An electron in a TV tube is beamed horizontally at a speed of  $5 \times 10^6$  m/sec toward the face of the tube 40 cm away. About how far will the electron drop before it hits?
- Equal-range firing angles** What two angles of elevation will enable a projectile to reach a target 16 km downrange on the same level as the gun if the projectile's initial speed is 400 m/sec?
- Finding muzzle speed** Find the muzzle speed of a gun whose maximum range is 24.5 km.