

## Exercises 14.1

### Domain, Range, and Level Curves

In Exercises 1–4, find the specific function values.

- $f(x, y) = x^2 + xy^3$ 
  - $f(0, 0)$
  - $f(-1, 1)$
  - $f(2, 3)$
  - $f(-3, -2)$
- $f(x, y) = \sin(xy)$ 
  - $f\left(2, \frac{\pi}{6}\right)$
  - $f\left(-3, \frac{\pi}{12}\right)$
  - $f\left(\pi, \frac{1}{4}\right)$
  - $f\left(-\frac{\pi}{2}, -7\right)$
- $f(x, y, z) = \frac{x - y}{y^2 + z^2}$ 
  - $f(3, -1, 2)$
  - $f\left(1, \frac{1}{2}, -\frac{1}{4}\right)$
  - $f\left(0, -\frac{1}{3}, 0\right)$
  - $f(2, 2, 100)$
- $f(x, y, z) = \sqrt{49 - x^2 - y^2 - z^2}$ 
  - $f(0, 0, 0)$
  - $f(2, -3, 6)$
  - $f(-1, 2, 3)$
  - $f\left(\frac{4}{\sqrt{2}}, \frac{5}{\sqrt{2}}, \frac{6}{\sqrt{2}}\right)$

In Exercises 5–12, find and sketch the domain for each function.

- $f(x, y) = \sqrt{y - x - 2}$
- $f(x, y) = \ln(x^2 + y^2 - 4)$
- $f(x, y) = \frac{(x - 1)(y + 2)}{(y - x)(y - x^3)}$
- $f(x, y) = \frac{\sin(xy)}{x^2 + y^2 - 25}$
- $f(x, y) = \cos^{-1}(y - x^2)$
- $f(x, y) = \ln(xy + x - y - 1)$
- $f(x, y) = \sqrt{(x^2 - 4)(y^2 - 9)}$
- $f(x, y) = \frac{1}{\ln(4 - x^2 - y^2)}$

In Exercises 13–16, find and sketch the level curves  $f(x, y) = c$  on the same set of coordinate axes for the given values of  $c$ . We refer to these level curves as a contour map.

- $f(x, y) = x + y - 1$ ,  $c = -3, -2, -1, 0, 1, 2, 3$
- $f(x, y) = x^2 + y^2$ ,  $c = 0, 1, 4, 9, 16, 25$
- $f(x, y) = xy$ ,  $c = -9, -4, -1, 0, 1, 4, 9$
- $f(x, y) = \sqrt{25 - x^2 - y^2}$ ,  $c = 0, 1, 2, 3, 4$

In Exercises 17–30, (a) find the function's domain, (b) find the function's range, (c) describe the function's level curves, (d) find the boundary of the function's domain, (e) determine if the domain is an open region, a closed region, or neither, and (f) decide if the domain is bounded or unbounded.

- $f(x, y) = y - x$
- $f(x, y) = \sqrt{y - x}$
- $f(x, y) = 4x^2 + 9y^2$
- $f(x, y) = x^2 - y^2$

21.  $f(x, y) = xy$

22.  $f(x, y) = y/x^2$

23.  $f(x, y) = \frac{1}{\sqrt{16 - x^2 - y^2}}$

24.  $f(x, y) = \sqrt{9 - x^2 - y^2}$

25.  $f(x, y) = \ln(x^2 + y^2)$

26.  $f(x, y) = e^{-(x^2 + y^2)}$

27.  $f(x, y) = \sin^{-1}(y - x)$

28.  $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$

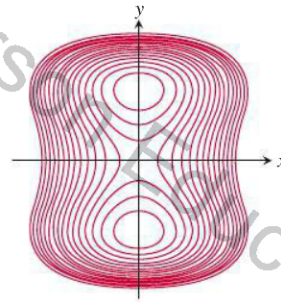
29.  $f(x, y) = \ln(x^2 + y^2 - 1)$

30.  $f(x, y) = \ln(9 - x^2 - y^2)$

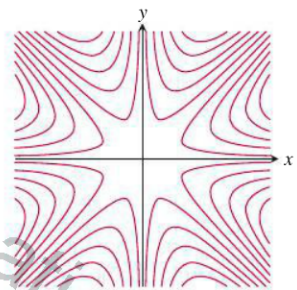
### Matching Surfaces with Level Curves

Exercises 31–36 show level curves for the functions graphed in (a)–(f) on the following page. Match each set of curves with the appropriate function.

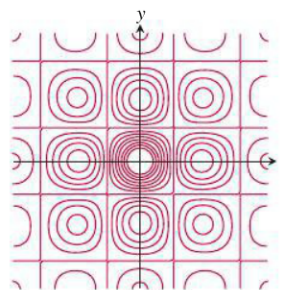
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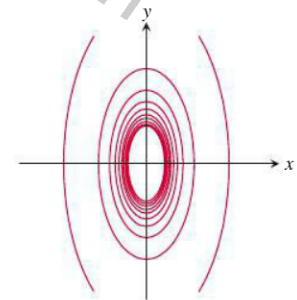
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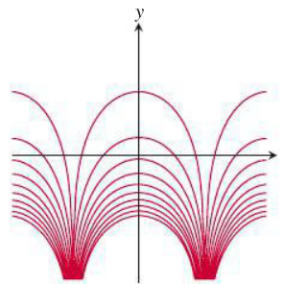
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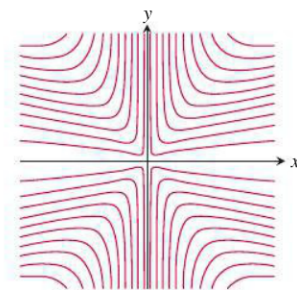
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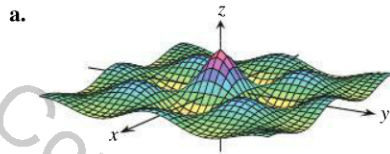


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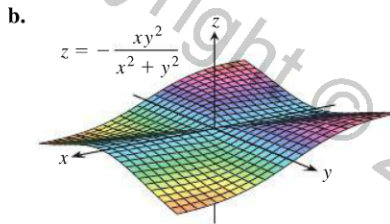


36.

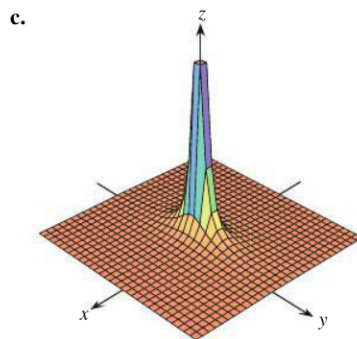




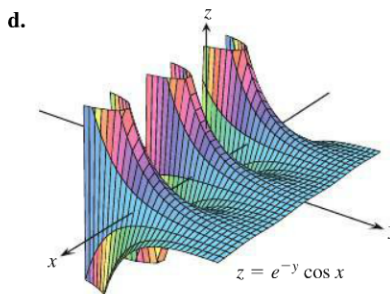
$$z = (\cos x)(\cos y) e^{-\sqrt{x^2+y^2}/4}$$



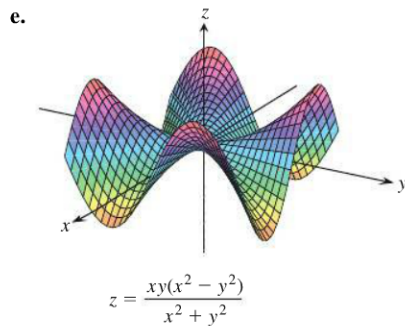
$$z = -\frac{xy^2}{x^2+y^2}$$



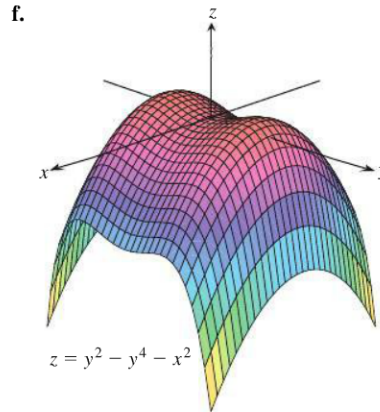
$$z = \frac{1}{4x^2+y^2}$$



$$z = e^{-y} \cos x$$



$$z = \frac{xy(x^2-y^2)}{x^2+y^2}$$



$$z = y^2 - y^4 - x^2$$

### Functions of Two Variables

Display the values of the functions in Exercises 37–48 in two ways: (a) by sketching the surface  $z = f(x, y)$  and (b) by drawing an assortment of level curves in the function's domain. Label each level curve with its function value.

- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| 37. $f(x, y) = y^2$                  | 38. $f(x, y) = \sqrt{x}$             |
| 39. $f(x, y) = x^2 + y^2$            | 40. $f(x, y) = \sqrt{x^2 + y^2}$     |
| 41. $f(x, y) = x^2 - y$              | 42. $f(x, y) = 4 - x^2 - y^2$        |
| 43. $f(x, y) = 4x^2 + y^2$           | 44. $f(x, y) = 6 - 2x - 3y$          |
| 45. $f(x, y) = 1 -  y $              | 46. $f(x, y) = 1 -  x  -  y $        |
| 47. $f(x, y) = \sqrt{x^2 + y^2 + 4}$ | 48. $f(x, y) = \sqrt{x^2 + y^2} - 4$ |

### Finding Level Curves

In Exercises 49–52, find an equation for and sketch the graph of the level curve of the function  $f(x, y)$  that passes through the given point.

49.  $f(x, y) = 16 - x^2 - y^2$ ,  $(2\sqrt{2}, \sqrt{2})$   
 50.  $f(x, y) = \sqrt{x^2 - 1}$ ,  $(1, 0)$   
 51.  $f(x, y) = \sqrt{x + y^2} - 3$ ,  $(3, -1)$   
 52.  $f(x, y) = \frac{2y - x}{x + y + 1}$ ,  $(-1, 1)$

### Sketching Level Surfaces

In Exercises 53–60, sketch a typical level surface for the function.

53.  $f(x, y, z) = x^2 + y^2 + z^2$     54.  $f(x, y, z) = \ln(x^2 + y^2 + z^2)$   
 55.  $f(x, y, z) = x + z$     56.  $f(x, y, z) = z$   
 57.  $f(x, y, z) = x^2 + y^2$     58.  $f(x, y, z) = y^2 + z^2$   
 59.  $f(x, y, z) = z - x^2 - y^2$   
 60.  $f(x, y, z) = (x^2/25) + (y^2/16) + (z^2/9)$

### Finding Level Surfaces

In Exercises 61–64, find an equation for the level surface of the function through the given point.

61.  $f(x, y, z) = \sqrt{x - y} - \ln z$ ,  $(3, -1, 1)$   
 62.  $f(x, y, z) = \ln(x^2 + y + z^2)$ ,  $(-1, 2, 1)$