

Homework 2, due Thursday, Sept 29

In section 1.2, please do problems 5, 6ab, 7, 8, 13, 17, 22. Also answer the following question:

A) Consider the initial value problem

$$u_t + uu_x = 0, \quad x \in \mathbb{R}, \quad t > 0$$

with

$$u(x, 0) = \begin{cases} 3 & \text{if } x \leq 0 \\ 3 - x & \text{if } 0 \leq x \leq 2. \\ 1 & \text{if } 2 \leq x \end{cases}$$

1. Draw the characteristics in the (x, t) plane.
2. Determine the time t_* at which a shock forms.
3. Find a formula for the solution for $0 \leq t \leq t_*$.

In section 1.3, please do problems 2, 3, 4, 6, 9, 10. In problems 4 and 6, you only need to solve for the steady state. (You can ignore the other parts of problems 4 and 6 as stated in the book.) Also note there is a typo in problem 10, and the initial equation should read

$$u_t = Du_{xx} - uu_x,$$

which gets transformed into the equation $v_t = Dv_{xx}$. In other words, the transformation removes the nonlinear term in the equation.