Academic conduct statement [3 points] Please write out the statement "I am aware that this exam, like any exam, is governed by the Boston University academic conduct code."

Please print your name:

Please sign your name:

Please write your BU ID number:

Midterm 2b - MA 225 - Fall 2016

## Thursday, November 10, 2016

Name: $\qquad$ BU ID: $\qquad$
Discussion section (circle one):
B2: W 9-10, B3: W 2-3, B4: W 1-2, B5: Th 830-930, B6: Th 930-1030

Instructions: Please write clearly and show all work. If an answer is not justified, no points will be awarded. Points may be deducted for messy, unclear, or poorly explained work. Books, notes, and calculators are NOT permitted during this exam.
$\qquad$

Do not write in the following box.

| Problem | Possible | Score |
| :---: | :---: | :---: |
| Academic Conduct Statement | 3 |  |
| Name, BU ID, discussion | 2 |  |
| 1 | 15 |  |
| 2 | 10 |  |
| 3 | 15 |  |
| 4 | 15 |  |
| 5 | 10 |  |
| 6 | 10 |  |
| 7 | 10 |  |
| 8 | 10 |  |
| Total | 100 |  |

## Question 1 [15 points]

(i) $[8$ points] Given

$$
f(x, y)=\sin \left(\frac{y}{x}\right)
$$

compute $f_{y}(x, y), f_{y y}(x, y)$, and $f_{y x}(x, y)$.
(ii) [7 points] Let $g(x, y)=y \ln (3 x+y), x(t, s)=t s$, and $y(t, s)=e^{s t^{2}}$. Compute $d g / d t$. Make sure your answer is in terms of $s$ and $t$ only.

## Question 2 [10 points]

(i) [5 points] Determine all values of $(x, y)$ where the following function is continuous:

$$
f(x, y)= \begin{cases}\frac{5 x+y-1}{x^{2}+4} & (x, y) \neq(0,0) \\ 2 & (x, y)=(0,0)\end{cases}
$$

Be sure to justify your answer.
(ii) [5 points] Evaluate the following limit or determine that it does not exist.

$$
\lim _{(x, y) \rightarrow(0,0)} \frac{x-y^{2}}{x-y}
$$

## Question 3 [15 points]

(i) [8 points] Determine the domain and range of the following function. Please be sure to justify your answer.

$$
f(x, y)=10 \ln \left(y-x^{2}-2\right)
$$

(ii) [7 points] Sketch the level curves of the following function.

$$
f(x, y)=5 e^{4-x^{2}-y^{2}}
$$

## Question 4 [15 points]

(i) [5 points] Compute the directional derivative of the following function at the given point in the direction of the given vector.

$$
h(x, y)=23+x y^{2}+\cos y, \quad P(3,1), \quad\langle 5,2\rangle .
$$

(ii) [5 points] In what direction does the function $h(x, y)$ from part (i) have the greatest decrease?
(iii) [5 points] The derivative of a function $f(x, y)$ at a point $P$ is greatest in the direction of the vector $\langle 1,-1\rangle$. In this direction, the value of its derivative is $5 \sqrt{2}$. What is $\nabla f$ at $P$ ?

Question 5 [10 points] Find the local maxima, minima, and saddle points of $f(x, y)=x^{3}+y^{3}-$ $6 x^{2}+6 y^{2}-9$.

## Question 6 [10 points]

(i) [5 points] Find the equation of the tangent plane to the surface $z=y^{2}+x y-x^{2}$ at the point $(x, y)=(1,1)$.
(ii) [5 points] By about how much will $f(x, y, z)=e^{y} \cos (x z)$ change as the point $P(x, y, z)$ moves from the origin a distance of $d s=0.1$ unit in the direction of $\langle 3,1,-3\rangle$.

Question 7 [ 10 points] Consider the following integral

$$
\int_{0}^{9} \int_{-\sqrt{9-y}}^{(y-9) / 3} \frac{x^{3}}{1+y^{2}} \mathrm{~d} x \mathrm{~d} y
$$

(i) [5 points] Sketch the region of integration.
(ii) [5 points] Write down an equivalent integral, with the order of integration reversed. Make sure you clearly state the limits of integration. (Note, you do not need to evaluate the resulting integral.)

## Question 8 [10 points]

(i) [5 points] Evaluate the following integral using a method of your choice.

$$
\int_{0}^{2} \int_{0}^{y^{3}} e^{x / y} \mathrm{~d} x \mathrm{~d} y
$$

(ii) [5 points] Set up, but do not evaluate, a double integral that represents the volume of the solid that lies below the surface defined by $f(x, y)=5 x y$ and above the region in the $x y$-plane that lies between $y=x, x+y=4$, and the $x$-axis. Make sure you clearly state the limits of integration and order of integration.

