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**Academic conduct statement [3 points]** Please write out the statement “I am aware that this exam, like any exam, is governed by the Boston University academic conduct code.”

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Please print your name:

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Please sign your name:

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Please write your BU ID number:

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## Practice Final Exam – MA 225 – Fall 2016

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Name: \_\_\_\_\_ BU ID: \_\_\_\_\_

Discussion section (circle one):

B2: W 9-10, B3: W 2-3, B4: W 1-2, B5: Th 830-930, B6: Th 930-1030

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Instructions: Please write clearly and **show all work**. **If an answer is not justified, no points will be awarded.** Points may be deducted for messy, unclear, or poorly explained work. Books, notes, and calculators are NOT permitted during this exam.

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Do not write in the following box.

Problem	Possible	Score
Academic Conduct Statement	3	
Name, BU ID, discussion	2	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

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**Question 1 [10 points]** Evaluate

$$\iint_R \sin(9x^2 + 9y^2) dA$$

where  $R$  is the region in the first quadrant bounded by the circle  $x^2 + y^2 = 16$ .

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**Question 2 [10 points]**

- (i) [4 points] Suppose  $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$  is a vector and  $\mathbf{F} = \mathbf{F}(x, y, z)$  is a vector field in  $\mathbf{R}^3$ . Is the following quantity meaningful?

$$\mathbf{b} \times (\nabla \cdot \mathbf{F})$$

If so, is it a scalar or a vector? (Make sure you justify your answer.)

- (ii) [5 points] Are the following lines parallel?

$$\mathbf{r}_1(t) = \langle 1 + 2t, 3t, 4 - t \rangle, \quad \mathbf{r}_2(t) = \langle -5 + 4t, 7 + 6t, 1 - 2t \rangle, \quad -\infty < t < \infty$$

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**Question 3 [10 points]** Evaluate

$$\int_C xy dx + y dy,$$

where  $C$  is the curve given by  $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$  for  $0 \leq t \leq \pi/2$ .

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**Question 4 [10 points]**

Consider  $\iint_S \mathbf{F} \cdot \mathbf{n} d\sigma$  where  $\mathbf{F}(x, y, z) = \langle y, z - y, x \rangle$  and  $S$  is the surface of the tetrahedron with vertices  $(0, 0, 0)$ ,  $(1, 0, 0)$ ,  $(0, 1, 0)$ , and  $(0, 0, 1)$ . Write down, but do not evaluate, an equivalent triple integral. Be sure to specify the limits and order of integration in the triple integral.

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**Question 5 [10 points]**

(i) Evaluate the integral

$$\int_{-3}^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} (x^2z + y^2z + z^3) dz dx dy.$$

(ii) Determine the equation of the plane that contains the point  $(0, 1, 0)$  and does not intersect the  $xz$ -plane.



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**Question 6 [10 points]**

Determine whether or not the limit exists. If it exists, find its value. (Make sure to justify your answer.)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{3x^2 + 2y^2}$$

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**Question 7 [10 points]**

Sketch the solid whose volume is given by the integral

$$\int_0^1 \int_0^{1-x^2} (1-x) dy dx.$$

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**Question 8 [10 points]**

Determine the area of the surface consisting of the part of the paraboloid  $z = x^2 + y^2$  that lies inside the cylinder  $x^2 + y^2 = 9$ .

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**Question 9 [10 points]**

Show that the ellipsoid  $3x^2 + 2y^2 + z^2 = 9$  and the sphere  $x^2 + y^2 + z^2 - 8x - 6y - 8z + 24 = 0$  are tangent to each other at the point  $(1, 1, 2)$ . (This means they have the same tangent plane at that point.)

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**Question 10 [10 points]**

(i) Find  $f_z$  if

$$f(x, y, z) = e^{\sin(xyz)} + \frac{x^2y}{1+z^2}.$$

(ii) Find the directional derivative of  $f(x, y, z) = x^2yz$  at the point  $(1, 2, 3)$  in the direction of the vector  $\langle 1, 0, 1 \rangle$ .