Practice Midterm 1– MA 225 – Fall 2016

Name: ______________________________ BU ID: ______________________________

Discussion section (circle one):

B2: W 9-10, B3: W 2-3, B4: W 1-2, B5: Th 830-930, B6: Th 930-1030

Instructions: Please write clearly and **show all work. If an answer is not justified, no points will be awarded.** Points may be deducted for messy, unclear, or poorly explained work. Books, notes, and calculators are NOT permitted during this exam.

Do not write in the following box.

Problem	Possible	Score
Name, BU ID, discussion	2	
Academic Conduct Statement	3	
1	10	
2	20	
3	15	
4	10	
5	10	
6	10	
7	10	
8	10	
Total	100	

Academic conduct statement [3 points] Please write out the statement “I am aware that this exam, like any exam, is governed by the Boston University academic conduct code” and then sign your name.

Question 1 [10 points] Let

$$\mathbf{u} = \langle 2, -1, 3\rangle, \qquad \mathbf{v} = \langle 4, 5, 0\rangle, \qquad \mathbf{w} = \langle 1, -1, -4\rangle$$

be vectors in $\mathbb{R}^3$. For each of the following, if the quantity makes sense, compute it. If it does not make sense, explain why.

(i) $(2\mathbf{u}) \cdot \mathbf{v}$

(ii) $(\mathbf{u} - \mathbf{v}) \times \mathbf{w}$

(iii) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

(iv) $|\mathbf{w}|$

Question 2 [20 points] Let

$$\mathbf{u} = \langle 1, -5, 4 \rangle, \qquad \mathbf{v} = \langle -2, c, -8 \rangle, \qquad \mathbf{w} = \langle 3, 1, 0 \rangle$$

be vectors in $\mathbb{R}^3$, where c is a scalar.

(i) Find two unit vectors that are parallel to $\mathbf{u}$.

(ii) Find a value of c such that $\mathbf{u}$ and $\mathbf{v}$ are parallel.

(iii) Find a value of c such that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

(iv) Compute $\mathrm{proj}_{\mathbf{u}}\mathbf{w}$.

Question 3 [15 points]

(i) [**5 points**] Give a geometric description of the set of points (x, y, z) that satisfy

$$x^2 + y^2 + z^2 - 4x + 6z \geq 10.$$

(ii) [**5 points**] Write down an equation describing the plane that is parallel to the xz-plane and that contains the point $(-3, 2, -8)$.

(iii) [**5 points**] Describe the set of all vectors whose projection onto the unit coordinate vector $\mathbf{k}$ is zero, and draw a picture of the collection of all such vectors.

Question 4 [10 points]

(i) [**5 points**] Find an equation of the line containing the points $(0, 2, 3)$ and $(1, -4, 2)$.

(ii) [**5 points**] Sketch the curve described by the following function and describe in words all key aspects of your picture.

$$\mathbf{r}(t) = \langle 2\cos 3t, e^t, 2\sin 3t \rangle, \qquad -\infty < t < \infty$$

Question 5 [10 points]

(i) **[5 points]** A fish in the water is climbing at an angle of 60 degrees above the horizontal with speed 2 mi/hr. Assuming the motion takes place within a two dimensional plane, find the two components of its velocity vector.

(ii) **[5 points]** Suppose a projectile begins at the point $(0, 3, 4)$ with an initial velocity vector of $\langle 1, 2, 3 \rangle$. If its acceleration is given by

$$\mathbf{a}(t) = \langle t, e^{-t}, 2 \rangle,$$

find the velocity and position vectors for $t \geq 0$.

Question 6 [10 points]

(i) [**5 points**] Find the equation of the plane that contains the following point and line.

$$(-2, 1, 4), \qquad \mathbf{r}_1(t) = \langle 1, 2, 3\rangle + t\langle -1, 0, 4\rangle, \quad -\infty < t < \infty$$

(ii) [**5 points**] Find the equation of the tangent line to the curve $\mathbf{r}(t) = \langle e^t, t^4, t^{-1}\rangle$ at the point $(e, 1, 1)$.

Question 7 [10 points] Consider the surface described by the equation

$$9z^2 + \frac{x^2}{4} - y^2 - 1 = 0.$$

Sketch the traces in the three coordinate planes. Then sketch the complete surface in $\mathbb{R}^3$. Please be sure to justify your answer.

Question 8 [10 points]

(i) **[5 points]** Compute the length of the curve $\mathbf{r}(t) = \langle 2t^3, -t^3, 2t^3 \rangle$ between the points $(2, -1, 2)$ and $(16, -8, 16)$.

(ii) **[5 points]** Consider a point $S(1, 0, c)$ for some scalar c and consider the line $\mathbf{r}(t) = \langle 1, 2, 0 \rangle + t\langle -1, 0, 1 \rangle$. Find a value of c such that the distance from the point to the line is equal to 4.