**Homework Assignment 2, Due Date Monday, February 8**

1) Assume that $f : X \to X$ and $g : Y \to Y$ are semi-conjugate, so there exists a continuous and onto map $h : X \to Y$ such that $h \circ f = g \circ h$.

   (a) If $p$ is a periodic point for $f$ with periodic $n$, prove that $h(p)$ is a periodic point for $g$ whose period divides $n$. If $h$ is a conjugacy, prove their periods coincide.

   (b) If $p$ is a stable fixed point for $f$, prove $h(p)$ is a stable fixed point for $g$.

   (c) Prove that if $f$ is topologically transitive, then so is $g$.

   (d) Prove that if $f$ is topologically mixing, then so is $g$.

2) Consider the following maps

$$\begin{aligned}
E_2 : &\quad S^1 \to S^1, &\quad E_2(z) = z^2, \\
g(y) : &\quad [-1,1] \to [-1,1], &\quad g(y) = 2y^2 - 1, \\
q_4(x) : &\quad [0,1] \to [0,1], &\quad q_4(x) = 4x(1-x),
\end{aligned}$$

and

$$T : [0,1] \to [0,1] \quad T(w) = \begin{cases} 2w & \text{if } 0 \le w \le 1/2 \\ 2 - 2w & \text{if } 1/2 \le w \le 1. \end{cases}$$

   (a) Prove that $E_2$ is semi-conjugate to both $T$ and $g$.

   (b) Prove that $T$, $g$, and $q_4$ are all conjugate.

   (Hint: consult Chapter 2 of Milnor for ideas on what maps to use to construct the conjugacies).

3) Verify the statements made on page 62 of Milnor's book after the proof of Lemma 4.2.6. In other words, given

$$x \to f(x) = x + \frac{1}{5}\sin^2(\pi x) \text{ mod } 1,$$

   prove that the only nonwandering point is 0, but that every point is chain recurrent.

4) Suppose that $f : X \to X$ is a continuous map on a compact metric space. Suppose further that it is "locally distance increasing," meaning that for all $x \neq y$ with $d(x,y) < 2\epsilon$, $d(f(x), f(y)) > d(x,y)$. Prove that, for any $x \neq y$, there exists some $N = N(x,y) \geq 0$ such that $d(f^N(x), f^N(y)) > \epsilon$.

5) An $\epsilon$-pseudo orbit for a map $f$ on a metric space $X$ with distance $d$ is a sequence of points $y_0, \dots, y_n$ such that $d(f(y_i), y_{i+1}) < \epsilon$ for all $0 \le i < n$. The map $f$ is said to have the shadowing property if for every such orbit there is a true orbit $x_0, \dots, x_n = f^n(x_0)$ with $d(x_i, y_i) < \epsilon'$ for all $0 \le i \le n$, where $\epsilon' \to 0$ as $\epsilon \to 0$.

   Show that the doubling map $E_2$ on the circle has the shadowing property. (Take $x_n = y_n$ and work backwards.) Conclude that the quadratic map $g(x) = 2x^2 - 1$ on the interval $[-1,1]$ does also. On the other hand, show that a rotation of the circle does not.

6) Suppose $X$ is compact and $f : X \to X$ is a homeomorphism. If $U$ is a neighborhood of $\Omega(f)$ and $x \in X$, show there exists an integer $N$ such that $f^n(x) \in U$ for all $n \geq N$.