Homework Assignment 3, Due Date Wednesday, February 24

1) Prove that if f is semi-conjugate to g ($g \circ h = h \circ f$ where h is continuous and onto), then $h(g) \leq h(f)$.

2) Prove that if A is an eventually positive transition matrix then $\sigma_A : \Sigma_A^+ \to \Sigma_A^+$ is topologically mixing and has dense periodic orbits. (Since minimality implies transitivity, this means the map is chaotic. Hint: first show that A^m positive for some m implies A^k positive for all $k \geq m$.)

3) Compute the topological entropy of the following maps:

   (a) A diffeomorphism of the circle. (An irrational rotation would be an example of this.)

   (b) $f(x) = x(1-x)$ on $[0,1]$

   (c) The maps $g_{1,2} : [0,1] \to [0,1]$, where

$$g_1(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1/3 \\ 1-x & \text{if } 1/3 \leq x \leq 2/3 \\ 2x-1 & \text{if } 2/3 \leq x \leq 1 \end{cases} \qquad g_2(x) = \begin{cases} 2/3-2x & \text{if } 0 \leq x \leq 1/3 \\ 3x-1 & \text{if } 1/3 \leq x \leq 2/3 \\ 7/3-2x & \text{if } 2/3 \leq x \leq 1 \end{cases}$$

4) Suppose $f : [0,1] \to [0,1]$ is continuous and there are two closed intervals $I_1, I_2 \subset [0,1]$ (disjoint except maybe at their endpoints) such that $I_1 \cup I_2 \subset f(I_1)$, and $I_1 \cup I_2 \subset f(I_2)$. Prove $h(f) \geq \log 2$.

5) (a) Let $f : \mathbb{R} \to \mathbb{R}$ be C^1. Suppose p is a periodic point and $\omega(x_0) = \mathcal{O}(p)$. Prove $\lambda(x_0) = \lambda(p)$.

   (b) Compute the Lyapunov exponents of $q_\alpha(x) = \alpha x(1-x)$ for (i) $1 < \alpha < 3$ and (ii) $3 < \alpha < 1 + \sqrt{6}$.