Midterm 1a – MA 225 B1 – Spring 2011

Instructor: Margaret Beck
TF: Man-Ho Ho
Date: February 15, 2011

Name: ______________________________

BU ID: ______________________________

Score:	1.	____________	(out of 10)
	2.	____________	(out of 10)
	3.	____________	(out of 10)
	4.	____________	(out of 10)
	5.	____________	(out of 20)
	6.	____________	(out of 15)
	7.	____________	(out of 20)
	8.	____________	(out of 5)
Total:		____________	(out of 100)

Instructions: Please write clearly and show all work. No credit will be given if answers are not justified. If the problem asks you to use a specific method in your solution, please make sure you do so. No credit will be given if another method is used. The point value of each problem is written in bold at the beginning of each problem. If you have any questions, please ask!

Question 1 [10 points] State whether the following expressions are meaningful. If they are, state whether they are a scalar or a vector. Note that $\mathbf{a}, \mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ are all vectors, $\times$ denotes cross product, and $\cdot$ denotes dot product. (Please justify your answers.)

(i) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$.

(ii) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$.

Question 2. [10 points] Find the equation of the plane that passes through the point $P(1, -1, 2)$ and contains the line with symmetric equation $x = 2y = 3z$.

Question 3. [10 points] Sketch the solid described by the inequalities

$$1 \le \rho \le 4, \qquad \frac{\pi}{2} \le \phi \le \pi.$$

Question 4 [10 points] Reparameterize the following curve with respect to arc length from the point $t = 0$ in the direction of increasing t.

$$\mathbf{r}(t) = \langle e^t,\ e^t \sin t,\ e^t \cos t \rangle.$$

Question 5 [20 points] Consider the space curve defined by

$$\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + 2\mathbf{k}.$$

(i) Sketch the curve, and indicate with an arrow the direction in which t increases.

(ii) Determine the equation for the tangent line to the curve at $(4, -2, 2)$.

Question 6 [15 points] A gun is fired with angle of elevation $\pi/6$. What is the initial speed if the maximum height of the shell is $500m$? Recall that the equation for the motion of a projectile is

$$\mathbf{r}(t) = \langle (v_0 \cos\alpha)t,\ (v_0 \sin\alpha)t - \frac{1}{2}gt^2 \rangle,$$

where the initial velocity is given by $\mathbf{v}_0 = \langle (v_0 \cos\alpha), (v_0 \sin\alpha) \rangle$ and g is the gravitational constant. (Your answer may contain g - you don't need to know its value.)

Question 7 [20 points]

(i) Find a vector function that represents the curve of intersection of the following two surfaces:

$$z = 4x^2 + y^2, \qquad y = x^2.$$

(ii) Identify the surface with vector equation

$$\mathbf{r}(s,t) = \langle s\sin 2t,\ s^2,\ s\cos 2t\rangle.$$

Question 8 [5 points] If $\mathbf{a} = \langle 2, 1, 3 \rangle$ and $\mathbf{b} = \langle -1, 0, 1 \rangle$, find the vector projection of $\mathbf{a}$ onto $\mathbf{b}$, $\text{proj}_{\mathbf{b}}\mathbf{a}$.

Midterm 1b – MA 225 B1 – Spring 2011

Instructor: Margaret Beck
TF: Man-Ho Ho
Date: February 15, 2011

Name: ______________________________

BU ID: ______________________________

Score:	1.	______________	(out of 5)
	2.	______________	(out of 10)
	3.	______________	(out of 20)
	4.	______________	(out of 10)
	5.	______________	(out of 10)
	6.	______________	(out of 20)
	7.	______________	(out of 15)
	8.	______________	(out of 10)
Total:		______________	(out of 100)

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Question 1 [5 points] If $\mathbf{a} = \langle 3, 0, -2 \rangle$ and $\mathbf{b} = \langle 2, 1, 0 \rangle$, find the vector projection of $\mathbf{a}$ onto $\mathbf{b}$, $\mathrm{proj}_{\mathbf{b}}\mathbf{a}$.

Question 2. [10 points] Find the equation of the plane that passes through the point $P(2, 0, -1)$ and contains the line with symmetric equation $2x = y = 4z$.

Question 3 [20 points] Consider the space curve defined by

$$\mathbf{r}(t) = 2\mathbf{i} + t\mathbf{j} + t^2\mathbf{k}.$$

(i) Sketch the curve, and indicate with an arrow the direction in which t increases.

(ii) Determine the equation for the tangent line to the curve at $(2, -3, 9)$.

Question 4. [10 points] Sketch the solid described by the inequalities

$$2 \leq \rho \leq 5, \qquad 0 \leq \phi \leq \pi/2.$$

Question 5 [10 points] Reparameterize the following curve with respect to arc length from the point $t = 0$ in the direction of increasing t.

$$\mathbf{r}(t) = \langle e^t \sin t,\ e^t,\ e^t \cos t \rangle.$$

Question 6 [20 points]

(i) Find a vector function that represents the curve of intersection of the following two surfaces:

$$z = 2x^2 + 4y^2, \qquad x = y^2.$$

(ii) Identify the surface with vector equation

$$\mathbf{r}(s,t) = \langle t^2,\ t\sin 2s,\ t\cos 2s\rangle.$$

Question 7 [15 points] A gun is fired, and the maximum height of the shell is $200m$. If the angle of elevation $\pi/4$, what is the initial speed? Recall that the equation for the motion of a projectile is

$$\mathbf{r}(t) = \langle (v_0 \cos\alpha)t,\ (v_0 \sin\alpha)t - \frac{1}{2}gt^2 \rangle,$$

where the initial velocity is given by $\mathbf{v}_0 = \langle (v_0 \cos\alpha), (v_0 \sin\alpha) \rangle$ and g is the gravitational constant. (Your answer may contain g - you don't need to know its value.)

Question 8 [10 points] State whether the following expressions are meaningful. If they are, state whether they are a scalar or a vector. Note that $\mathbf{a}, \mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ are all vectors, $\times$ denotes cross product, and $\cdot$ denotes dot product. (Please justify your answers.)

(i) $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$.

(ii) $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d})$.