Instructions: Please write clearly and show all work. No credit will be given if answers are not justified. If the problem asks you to use a specific method in your solution, please make sure you do so. No credit will be given if another method is used. The point value of each problem is written in bold at the beginning of each problem. If you have any questions, please ask!
Question 1 [10 points] Use implicit differentiation to find $\frac{\partial z}{\partial y}$ if

$$yz = \ln(x + z).$$

---

Question 2. [10 points] Consider the function $g(x, y) = x^2 ye^{xy}$. Use its linearization at the point $(x, y) = (1, 1)$ to approximate the value of $g(0.9, 1.1)$. 
Question 3. [10 points] Find the absolute maximum and minimum values of the function \( f(x, y) = 4x + 6y - x^2 - y^2 \) on the set \( D = \{(x, y) \mid 0 \leq x \leq 4, \ 0 \leq y \leq 5\} \).
**Question 4 [10 points]** Find the maximum rate of change of \( g(x, y) = xe^{-y} + ye^{-x} \) at the point \((x, y) = (0, 0)\) and the direction in which it occurs.

**Question 5 [10 points]** Set up, but do not evaluate, the integral that represents the surface area of

\[
\mathbf{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + v \mathbf{k}, \quad 0 \leq u \leq 1, \quad 0 \leq v \leq \pi.
\]

(Make sure you determine explicitly what the integrand should be.)
Question 6 [10 points] Use the method of Lagrange multipliers to maximize the function \( f(x, y) = e^{xy} \) subject to the constraint \( x^3 + y^3 = 16 \).

Question 7 [10 points] Evaluate the following double integral:

\[
\int_0^1 \int_0^{x^2} \frac{y}{1 + x^5} \, dy \, dx.
\]
Question 8 [10 points] Set up, but do not evaluate, a double integral that represents the volume of the solid bounded by the planes $z = x$, $y = x$, $x + y = 2$, and $z = 0$.

Question 9 [10 points] Interchange the order of integration in the following integral. (You do not need to evaluate the resulting integral.)

$$
\int_{0}^{\sqrt{\pi}} \int_{y}^{\sqrt{\pi}} \cos(x^2) \, dx \, dy.
$$
Question 10 [10 points] Convert the following integral to polar coordinates. (You do not need to evaluate the resulting integral.)

\[ \int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} \, dy \, dx. \]
Instructions: Please write clearly and show all work. No credit will be given if answers are not justified. If the problem asks you to use a specific method in your solution, please make sure you do so. No credit will be given if another method is used. The point value of each problem is written in bold at the beginning of each problem. If you have any questions, please ask!
Question 1. [10 points] Evaluate the following double integral.

\[ \int_0^1 \int_0^{y^2} \frac{3x}{1+y^5} \, dx \, dy. \]

Question 2. [10 points] Interchange the order of integration in the following integral. (You do not need to evaluate the resulting integral.)

\[ \int_0^{\sqrt{\pi}} \int_x^{\sqrt{\pi}} \cos(y^2) \, dy \, dx. \]
Question 3 [10 points] Use the method of Lagrange multipliers to maximize the function $f(x, y) = e^{xy}$ subject to the constraint $x^3 + y^3 = 54$.

Question 4 [10 points] Find the maximum rate of change of $g(x, y) = xe^{-y} - ye^{-x}$ at the point $(x, y) = (1, 1)$ and the direction in which it occurs.
Question 5. [10 points] Consider the function \( g(x, y) = y^2xe^{yx} \). Use its linearization at the point \((x, y) = (1, 1)\) to approximate the value of \( g(0.9, 1.1) \).

Question 6 [10 points] Use implicit differentiation to find \( \partial z/\partial x \) if

\[
xz = \ln(y + z).
\]
Question 7 [10 points] Find the absolute maximum and minimum values of the function \( f(x, y) = 6x + 4y - x^2 - y^2 \) on the set \( D = \{(x, y) \mid 0 \leq x \leq 5, \: 0 \leq y \leq 4\} \).
Question 8 [10 points] Set up, but do not evaluate, a double integral that represents the volume of the solid bounded by the planes $z = x$, $y = x$, $x + y = 4$, and $z = 0$.

Question 9 [10 points] Convert the following integral to polar coordinates. (You do not need to evaluate the resulting integral.)

$$
\int_0^2 \int_0^{\sqrt{2y-y^2}} \sqrt{x^2 + y^2} \, dx \, dy.
$$
Question 10 [10 points] Set up, but do not evaluate, the integral that represents the surface area of

\[ \mathbf{r}(u, v) = u \mathbf{i} + v \sin u \mathbf{j} + v \cos u \mathbf{k}, \quad 0 \leq u \leq \pi, \quad 0 \leq v \leq 1. \]

(Make sure you determine explicitly what the integrand should be.)