Practice Final Exam – MA 225 B1 – Spring 2011

Score:	1.	________	(out of 8)
	2.	________	(out of 4)
	3.	________	(out of 7)
	4.	________	(out of 5)
	5.	________	(out of 9)
	6.	________	(out of 7)
	7.	________	(out of 5)
	8.	________	(out of 9)
	9.	________	(out of 5)
	10.	________	(out of 7)
	11.	________	(out of 8)
	12.	________	(out of 9)
	13.	________	(out of 5)
	14.	________	(out of 5)
	15.	________	(out of 7)
Total:		________	(out of 100)

IMPORTANT: This practice exam does not cover all possible topics that may be on the exam. Do not be surprised/upset if there is a question on the actual exam that pertains to material not covered by this practice final. For the actual final, you are responsible for all material covered in this course.

Study suggestion: Do not look at this exam until you have "finished" studying. At that point, go somewhere no one will bother you for two hours, and take this exam as if it is the real exam. (Do not mess around with the internet, your phone, ipod, etc, while taking this practice exam.) Grade yourself, or better yet trade with a friend and grade each other. (If your friend cannot follow your work, then you have not explained yourself very well - this means your are in danger of not receiving partial/full credit for something you think you understood.) Then go back and re-study the topics you wish to improve.

(i) [**8 points**] Use an appropriate change of variables to evaluate

$$\iint_R \sin(9x^2 + 4y^2)\,\mathrm{d}A$$

where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.

(ii) [**4 points**] Suppose $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is a vector and $\mathbf{F} = \mathbf{F}(x.y.z)$ is a vector field in $\mathbf{R}^3$. Is the following quantity meaningful?

$$\mathbf{b} \times (\mathrm{div}\mathbf{F})$$

If so, is it a scalar or a vector? (Make sure you justify your answer.)

(iii) [**7 points**] Find a parametric representation for the part of the plane $z = x + 3$ that lies on the cylinder $x^2 + y^2 = 1$.

(iv) [**5 points**] Determine whether the following two lines parallel, skew, or intersecting.

$$L_1 : x = 1 + 2t, \quad y = 3t, \quad z = 4 - t, \qquad\qquad L_2 : \frac{x-3}{2} = \frac{y+1}{1} = \frac{z-1}{7}$$

(v) [**9 points**] Evaluate $\iint_S \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle y, z - y, x \rangle$ and S is the surface of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, by transforming it into a triple integral using a theorem from class and evaluating that triple integral.

(vi) [**7 points**] Evaluate the integral

$$\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}}(x^2z+y^2z+z^3)\mathrm{d}z\mathrm{d}x\mathrm{d}y.$$

(vii) [**5 points**] Determine the equation of the plane that contains the point $(0,1,0)$ and does not intersect the xz-plane.

(viii) [**9 points**] Determine whether or not the limit exists. If it exists, find its value. (Make sure to justify your answer.)

$$\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^4 y}{3x^2 + 2y^2}$$

(ix) [**5 points**] Sketch the solid whose volume is given by the integral

$$\int_0^1 \int_0^{1-x^2} (1-x)\mathrm{d}y\mathrm{d}x.$$

(x) [**7 points**] Determine the area of the surface consisting of the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.

(xi) [**8 points**] Show that the ellipsoid $3x^2+2y^2+z^2 = 9$ and the sphere $x^2+y^2+z^2-8x-6y-8z+24 = 0$ are tangent to each other at the point $(1, 1, 2)$. (This means they have a common tangent plane at that point.)

(xii) [**9 points**] Evaluate $\iint_S y^2 \mathrm{d}S$ where S is the part of the sphere $x^2 + y^2 + z^2 = 2$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

(xiii) [**5 points**] Find f_z if

$$f(x, y, z) = e^{\sin(xyz)} + \frac{x^2 y}{1 + z^2}.$$

(xiv) **[5 points]** Find the directional derivative of $f(x, y, z) = x^2yz$ at the point $(1, 2, 3)$ in the direction of the vector $\langle 1, 0, 1\rangle$.

(xv) **[7 points]** Two legs of a right traingle are measured as 5m and 12m, with a possible error in measurement of at most 0.2m in each. Use differentials to estimate the maximum error in the calculation value of the length of the hypotenuse.