Solutions to Quiz 2 – MA 225 B2 – Spring 2011

Question 1 If we test the approach to the origin along a line of the form $y = mx$, we find

$$\frac{5mx^4}{(1+3m^4)x^4} = \frac{5m}{(1+3m^4)},$$

which is different for different values of m. Hence, the limit does not exist.

Question 2 Note that $x(t=3) = 1$ and $y(t=3) = 3e^3$. We then compute

$$\begin{aligned} \frac{dg}{dt} &= \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt} \\ &= \left[2x\cos(x+y) - x^2\sin(x+y)\right]\left(-\frac{3}{t^2}\right) - x^2\sin(x+y)(e^t + te^t) \\ &= -\frac{1}{3}[2\cos(1+3e^3) - \sin(1+3e^3)] - 4e^3\sin(1+3e^3). \end{aligned}$$

Question 3

(i) Note that this vector is not a unit vector. A unit vector pointing in the same direction is given by $\mathbf{u} = (1/\sqrt{5})\langle 1, -2\rangle$. Also, $\nabla f = \langle 2y+2x, 2x\rangle$, and so

$$D_{\mathbf{u}}f = \langle 2y+2x, 2x\rangle \cdot (1/\sqrt{5})\langle 1, -2\rangle = \frac{2}{\sqrt{5}}[y-x].$$

(ii) The equation for a tangent plane is $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$. Using the partial derivatives compute in part (i), we find $f_x(1,1) = 4$ and $f_y(1,1) = 2$. Hence, the plane is

$$z - 3 = 4(x-1) + 2(y-1).$$

(iii) A critical point occurs when both $f_x = 0$ and $f_y = 0$. Hence, we need $2(y+x) = 0$ and $2x = 0$, which holds only for $(0,0)$. Thus, the origin is the only critical point. We compute

$$D = f_{xx}f_{yy} - f_{xy}^2 = 2(0) - 2^2 = -4 < 0.$$

Thus, this point is a saddle.

Question 4 The equation for the area A is $A = xy$. Thus,

$$dD = ydx + xdy = 10(3) + 20(3) = 90\text{cm}^2.$$

Solutions to Quiz 2 – MA 225 B3 – Spring 2011

Question 1 If we test the approach to the origin along a line of the form $y = mx$, we find

$$\frac{3m^3x^4}{(2+m^4)x^4} = \frac{3m^3}{(2+m^4)},$$

which is different for different values of m. Hence, the limit does not exist.

Question 2 Note that $x(t = 2) = 1$ and $y(t = 2) = 2e^2$. We then compute

$$\begin{aligned} \frac{dg}{dt} &= \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt} \\ &= \left[y^2\cos(x+y)\right]\left(-\frac{2}{t^2}\right) + \left[2y\sin(x+y) + y^2\cos(x+y)\right](e^t + te^t) \\ &= -2e^4\cos(1+2e^2) + 3e^2[4e^2\sin(1+2e^2) + 4e^4\cos(1+2e^2)]. \end{aligned}$$

Question 3

(i) Note that this vector is not a unit vector. A unit vector pointing in the same direction is given by $\mathbf{u} = (1/\sqrt{5})\langle 1, -2\rangle$. Also, $\nabla f = \langle 2y, 2x + 2y\rangle$, and so

$$D_{\mathbf{u}}f = \langle 2y, 2x+2y\rangle \cdot (1/\sqrt{5})\langle 1, -2\rangle = \frac{2}{\sqrt{5}}[-2x - y].$$

(ii) The equation for a tangent plane is $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$. Using the partial derivatives compute in part (i), we find $f_x(1, 1) = 2$ and $f_y(1, 1) = 4$. Hence, the plane is

$$z - 3 = 2(x - 1) + 4(y - 1).$$

(iii) A critical point occurs when both $f_x = 0$ and $f_y = 0$. Hence, we need $2(y + x) = 0$ and $2y = 0$, which holds only for $(0, 0)$. Thus, the origin is the only critical point. We compute

$$D = f_{xx}f_{yy} - f_{xy}^2 = 2(0) - 2^2 = -4 < 0.$$

Thus, this point is a saddle.

Question 4 The equation for the area A is $A = xy$. Thus,

$$dA = ydx + xdy = 5(2) + 10(2) = 30\text{cm}^2.$$

Solutions to Quiz 2 – MA 225 B4 – Spring 2011

Question 1 If we test the approach to the origin along a line of the form $y = mx$, we find

$$\frac{2m^2x^4}{(1+5m^6x^2)x^4} = \frac{2m^2}{(1+5m^6x^2)} \to 2m^2 \text{ as } x \to 0,$$

which is different for different values of m. Hence, the limit does not exist.

Question 2 Note that $x(t=2) = \cos 2$ and $y(t=2) = 1$. We then compute

$$\begin{aligned}\frac{dg}{dt} &= \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt} \\ &= \left[2xe^{x+y} + x^2e^{x+y}\right](-\sin t) + \left[x^2e^{x+y}\right](-2/t^2) \\ &= -\sin 2(2\cos 2 + \cos^2 2)e^{1+\cos 2} - \frac{1}{2}\cos^2 2e^{1+\cos 2}.\end{aligned}$$

Question 3

(i) Note that this vector is not a unit vector. A unit vector pointing in the same direction is given by $\mathbf{u} = (1/\sqrt{10})\langle 3, -1\rangle$. Also, $\nabla f = \langle 2x+y, x+2y\rangle$, and so

$$D_{\mathbf{u}}f = \langle 2x+y, x+2y\rangle \cdot (1/\sqrt{10})\langle 3,-1\rangle = \frac{1}{\sqrt{10}}[5x+y].$$

(ii) The equation for a tangent plane is $z - z_0 = f_x(x_0, y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$. Using the partial derivatives compute in part (i), we find $f_x(1,1) = 3$ and $f_y(1,1) = 3$. Hence, the plane is

$$z - 3 = 3(x-1) + 3(y-1).$$

(iii) A critical point occurs when both $f_x = 0$ and $f_y = 0$. Hence, we need $2x+y=0$ and $2y+x=0$, which holds only for $(0,0)$. Thus, the origin is the only critical point. We compute

$$D = f_{xx}f_{yy} - f_{xy}^2 = 2(2) - 1^2 = 3 > 0, \qquad f_{xx} = 2 > 0.$$

Thus, this point is a local minimum.

Question 4 The equation for the volume V is $V = \pi r^2 h$. Thus,

$$dV = 2\pi rhdr + \pi r^2 dh = 400\pi(2) + 100\pi(2) = 1000\pi\text{cm}^3.$$

Solutions to Quiz 2 – MA 225 B5 – Spring 2011

Question 1 If we test the approach to the origin along a line of the form $y = mx$, we find

$$\frac{3m^2x^4}{(4+m^6x^2)x^4} = \frac{3m^2}{(4+m^6x^2)} \to \frac{3}{4}m^2 \text{ as } x \to 0,$$

which is different for different values of m. Hence, the limit does not exist.

Question 2 Note that $x(t = 4) = \cos 4$ and $y(t = 4) = 1$. We then compute

$$\begin{aligned}\frac{dg}{dt} &= \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt} \\ &= \left[y^2e^{x+y}\right](-\sin t) + \left[2ye^{x+y} + y^2e^{x+y}\right](-4/t^2) \\ &= -\sin 4e^{1+\cos 4} - \frac{3}{4}e^{1+\cos 4}.\end{aligned}$$

Question 3

(i) Note that this vector is not a unit vector. A unit vector pointing in the same direction is given by $\mathbf{u} = (1/\sqrt{13})\langle 2, -3\rangle$. Also, $\nabla f = \langle 2x + 3y, 3x + 2y\rangle$, and so

$$D_{\mathbf{u}}f = \langle 2x + 3y, 3x + 2y\rangle \cdot (1/\sqrt{13})\langle 2, -3\rangle = \frac{13}{\sqrt{13}}x.$$

(ii) The equation for a tangent plane is $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$. Using the partial derivatives compute in part (i), we find $f_x(1, 1) = 5$ and $f_y(1, 1) = 5$. Hence, the plane is

$$z - 5 = 5(x - 1) + 5(y - 1).$$

(iii) A critical point occurs when both $f_x = 0$ and $f_y = 0$. Hence, we need $2x+3y = 0$ and $2y+3x = 0$, which holds only for $(0, 0)$. Thus, the origin is the only critical point. We compute

$$D = f_{xx}f_{yy} - f_{xy}^2 = 2(2) - 3^2 = -5 < 0.$$

Thus, this point is a saddle.

Question 4 The equation for the volume V is $V = \pi r^2 h$. Thus,

$$dV = 2\pi rhdr + \pi r^2dh = 100\pi(3) + 25\pi(3) = 375\pi\text{cm}^3.$$