Question 1 A parameterization of the line is
\[ r(t) = (1 - t)(0, 2, 3) + t(1, 6, 4) = (t, 2 + 4t, 3 + t), \quad 0 \leq t \leq 1. \]
Thus,
\[ \int_C f(x, y, z)ds = \int_0^1 f(r(t))|r'(t)|dt = \sqrt{18} \int_0^1 t^2(2 + 4t)(3 + t)dt = 63\sqrt{18}/10. \]

Question 2 This region is described by \(-1 \leq z \leq 1, -4 \leq y \leq -x^2, \) and \(-2 \leq x \leq 2). Therefore,
\[ V = \int_{-1}^1 \int_{-2}^2 \int_{-x^2}^{-4} 1dxdydz. \]

Question 3 If we try to evaluate this integral using the formula \( \int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(r(t)) \cdot r'(t)dt, \) we get something difficult to evaluate, so instead we use Green’s Theorem. Hence,
\[ \int_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R \left( \frac{2}{1 + (y/x)^2} - 1 + \frac{2y}{x^2 + y^2} \right) dA = -\iint_R 1dA, \]
which is just \(-1\) times the area of \( R. \) Since \( R \) is just a circle of radius 2, this area, and the value of the integral, is \(-4\pi.\)

Question 4 The integral is independent of path if the vector field is conservative, ie if there is a function \( f \) such that \( \mathbf{F} = \nabla f. \) This is true if \( P_y = Q_x, \) where \( F = \langle P, Q \rangle. \) Since \( P_y = e^y \) and \( Q_x = e^y, \) the vector field is conservative and the integral is path-independent.

Question 5 If we define \( u = x+y \) and \( v = y-x, \) then \( 2 \leq v \leq 4, \) \( 0 \leq u \leq 2 \) and \( |\partial(x, y)/\partial(u, v)| = 1/2). \) Therefore,
\[ \iint_R \frac{x + y}{x - y}dA = \int_2^4 \int_0^2 \frac{-u}{v^2}dudv = -\frac{1}{2}(2)(\ln 4 - \ln 2) = -(2 \ln 2 - \ln 2) = -\ln 2. \]
**Question 1** A parameterization of the line is

\[ \mathbf{r}(t) = (1 - t)(-1, 5, 0) + t(1, 6, 4) = (-1 + 2t, 5 + t, 4t), \quad 0 \leq t \leq 1. \]

Thus,

\[
\int_C f(x, y, z)ds = \int_0^1 f(\mathbf{r}(t))|\mathbf{r}'(t)|dt = 16\sqrt{21} \int_0^1 t^2(-1 + 2t)(5 + t)dt = 236\sqrt{21}/15.
\]

**Question 2** This region is described by \(0 \leq z \leq 4\), \(x^2 \leq y \leq 9\), and \(-3 \leq x \leq 3\). Therefore,

\[ V = \int_0^4 \int_{-3}^3 \int_{x^2}^9 1 \, dy \, dx \, dz. \]

**Question 3** If we try to evaluate this integral using the formula \( \int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)dt \), we get something difficult to evaluate, so instead we use Green’s Theorem. Hence,

\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA = \iint_R \left( \frac{2}{1 + (y/x)^2} - 1 + \frac{2y}{x^2 + y^2} \right) \, dA = \iint_R 1 \, dA,
\]

which is just \(-1\) times the area of \(R\). Since \(R\) is just a circle of radius 3, this area, and the value of the integral, is \(-9\pi\).

**Question 4** The integral is independent of path if the vector field is conservative, i.e., if there is a function \( f \) such that \( \mathbf{F} = \nabla f \). This is true if \( P_y = Q_x \), where \( F = \langle P, Q \rangle \). Since \( P_y = -e^{-x} \) and \( Q_x = -e^{-x} \), the vector field is conservative and the integral is path-independent.

**Question 5** If we define \( u = x + y \) and \( v = y - x \), then \( 0 \leq v \leq 2, 2 \leq u \leq 4 \) and \( |\partial(x, y)/\partial(u, v)| = 1/2 \). Therefore,

\[
\iint_R \frac{x - y}{x + y} \, dA = \int_0^2 \int_2^4 -\frac{v}{u} \, dudv = -\frac{1}{2} \left( \ln 4 - \ln 2 \right) = -(2 \ln 2 - \ln 2) = -\ln 2.
\]