

Homework Assignment 1, Due Monday, January 27

- 1) (Ex 1.9, Chicone) Find the solution of

$$\dot{x} = 1/x, \quad x(0) = x_0.$$

What is the maximal interval of existence? What happens at the edge of this interval?

- 2) (Ex 1.6 Chicone)

- (a) Find the general solution of

$$\frac{dy}{dx} = \frac{y}{(y+2)e^y - 2x}.$$

(Hint: it may be helpful to recall what an “exact equation” is.)

- (b) What is the difference between solving the above equation and solving the following system?

$$\begin{aligned}\frac{dx}{dt} &= 2x - (2+y)e^y \\ \frac{dy}{dt} &= -y\end{aligned}$$

(You do not actually need to solve the system.)

- 3) (Ex 1.14, Chicone 06) Suppose that $F : \mathbb{R} \rightarrow \mathbb{R}$ is a smooth, positive, periodic function with period $p > 0$.

- (a) Prove that if $x(t)$ solves $\dot{x} = F(x)$ and

$$T := \int_0^p \frac{1}{F(y)} dy,$$

then $x(t+T) - x(t) = p$ for all $t \in \mathbb{R}$. (Hint: consider $G(x) = \int_c^x (1/F(y)) dy$. What can you say about the function $g(y) = G(y+p) - G(y)$ and the value of $G(x(b)) - G(x(a))$?)

- (b) Is the same true if F is allowed to change sign?

- 4) (Ex 1.21, Chicone 06) Solve the ODE $dx/dt = t$ and show that the group property does not hold, ie

$$\phi(t+s, x_0) \neq \phi(t, \phi(s, x_0)).$$

Explain intuitively why the group property should hold for an autonomous equation but not a nonautonomous one.