Instructions: Please write clearly and show all work. If an answer is not justified, no points will be awarded. Points may be deducted for messy, unclear, or poorly explained work. Books, notes, and calculators are NOT permitted during this exam.

Do not write in the following box.

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<tr>
<th>Problem</th>
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Question 1 [20 points] Let

\[ u = \langle 3, 2, -1 \rangle, \quad v = \langle 0, 2, 1 \rangle, \quad w = \langle 5, -1, 1 \rangle \]

be vectors in \( \mathbb{R}^3 \). For each of the following, if the quantity makes sense, compute it. If it does not make sense, explain why.

(i) \( u \cdot (5v) \)

(ii) \( u \times (v - w) \)

(iii) \( u \cdot (v \cdot w) \)

(iv) \( |u| \)
Question 2 [20 points] Let

\[ \mathbf{u} = \langle 2, -3, 1 \rangle, \quad \mathbf{v} = \langle c, 6, -2 \rangle, \quad \mathbf{w} = \langle 0, 2, 5 \rangle \]

be vectors in \( \mathbb{R}^3 \), where \( c \) is a scalar.

(i) Compute \( \text{proj}_u \mathbf{w} \).

(ii) Find a value of \( c \) such that \( \mathbf{u} \) and \( \mathbf{v} \) are parallel.

(iii) Find a value of \( c \) such that \( \mathbf{u} \) and \( \mathbf{v} \) are orthogonal.

(iv) Find two unit vectors that are parallel to \( \mathbf{u} \).
Question 3 [18 points]

(i) Describe the set of all vectors whose projection onto the unit coordinate vector \( \mathbf{j} \) is zero, and draw a picture of the collection of all such vectors.

(ii) Write down an equation describing the plane that is parallel to the \( yz \)-plane and that contains the point \((2, 1, -7)\).

(iii) Give a geometric description of the set of points \((x, y, z)\) that satisfy

\[
x^2 + y^2 + z^2 + 8y - 2z \geq 4.
\]
Question 4 [20 points]

(i) Find an equation of the line containing the points \((1, -1, 2)\) and \((3, 1, 4)\).

(ii) Sketch the curve described by the following function and describe in words all key aspects of your picture.

\[ r(t) = 3 \cos 5t, 3 \sin 5t, e^{-t}, \quad -\infty < t < \infty \]
Question 5 [20 points]

(i) A dolphin in the water is climbing at an angle of 30 degrees above the horizontal with a heading to the northwest. If its speed is 5 mi/hr, find the three components of its velocity vector.

(ii) Suppose a projectile begins at the point (1, 0, 2) with an initial velocity vector of \(\langle 2, 1, 3 \rangle\). If its acceleration is given by

\[
a(t) = \langle e^{-t}, 5, 3t \rangle,
\]

find the velocity and position vectors for \(t \geq 0\).