Test 2A – MA 225 – Spring 2015

March 25, 2015

Name: ______________________________ BU ID: ______________________________

Discussion section (circle one):

A2: W 12-1, A3: W 3-4, A4: W 4-5, A5: Th 830-930, A6: Th 930-1030

Instructions: Please write clearly and **show all work. If an answer is not justified, no points will be awarded.** Points may be deducted for messy, unclear, or poorly explained work. Books, notes, and calculators are NOT permitted during this exam.

Do not write in the following box.

Problem	Possible	Score
Name, BU ID, discussion	2	
1	24	
2	20	
3	20	
4	18	
5	16	
Total	100	

Question 1 [24 points]

(i) **[8 points]** Let $g(x, y) = ye^{x^2}$, $x(t) = 2t^2$, and $y(t) = \sin t$. Compute dg/dt. Make sure your answer is in terms of t only.

(ii) **[8 points]** Compute the directional derivative of the following function at the given point in the direction of the given vector.

$$h(x, y) = 15 + 2x^2 - 4y^2, \qquad P(1, 4), \qquad \langle 1, 2 \rangle.$$

(iii) **[8 points]** For the above function $h(x, y)$ in part (ii), find a vector that points in a direction of no change in the function at $P(1, 4)$.

Question 2 [20 points]

(i) **[8 points]** Find the equation of the plane that contains the following point and line.

$$(-2, 1, 4), \qquad \mathbf{r}_1(t) = \langle 1, 2, 3 \rangle + t\langle -1, 0, 4 \rangle, \quad -\infty < t < \infty$$

(ii) **[12 points]** Find the point on the plane $2x + y - z = 6$ nearest the point $P(1, 2, 1)$. (Do not use Lagrange multipliers to solve this problem. Please use another method.)

Question 3 [20 points]

(i) [**8 points**] Determine the domain and range of the following function. Please be sure to justify your answer.

$$f(x, y) = 5e^{-\sqrt{4-x^2-y^2}}$$

(ii) [**12 points**] Sketch the level curves of the following function.

$$f(x, y) = 3\ln(x - 2y^2 + 4)$$

Question 4 [18 points]

(i) [**8 points**] True or false: if

$$\lim_{(x,0)\to(0,0)} f(x,0) = L, \qquad \lim_{(0,y)\to(0,0)} f(0,y) = L,$$

then $\lim_{(x,y)\to(0,0)} f(x,y)$ necessarily exists and is equal to L. Be sure to justify your answer.

(ii) [**10 points**] Evaluate the following limit or determine that it does not exist.

$$\lim_{(x,y)\to(0,1)} \frac{y\sin x}{x(y+1)}$$

Question 5 [16 points] Consider the surface described by the equation

$$9z^2 + \frac{x^2}{4} - y^2 - 1 = 0.$$

Sketch the traces in the three coordinate planes. Then sketch the complete surface in $\mathbb{R}^3$. Please be sure to justify your answer.