Test 3A – MA 225 – Spring 2015

April 24, 2015

Name: _____ BU ID: _____

Discussion section (circle one):

A2: W 12-1, A3: W 3-4, A4: W 4-5, A5: Th 830-930, A6: Th 930-1030

Instructions: Please write clearly and **show all work. If an answer is not justified, no points will be awarded.** Points may be deducted for messy, unclear, or poorly explained work. Books, notes, and calculators are NOT permitted during this exam.

Do not write in the following box.

Problem	Possible	Score
Name, BU ID, discussion	2	
1	20	
2	18	
3	20	
4	20	
5	20	
Total	100	

Question 1 [20 points] Evaluate the following two integrals using a method of your choice.

(i) **[10 points]**

$$\int_0^2 \int_0^{4-y} (x+y)\mathrm{d}x\mathrm{d}y$$

(ii) **[10 points]**

$$\int_C (x^2+y^2)\ \mathrm{d}s,$$

where C is the line segment from $(2,2)$ to $(8,8)$.

Question 2 [18 points] Consider

$$f(x,y,z) = xy\sin z, \qquad \mathbf{F}(x,y,z) = \langle x, y^2 z, x - y\rangle, \qquad C : \mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$$

(i) **[7 points]** Compute $\nabla \cdot \mathbf{F}$.

(ii) **[7 points]** Compute $\nabla \times \mathbf{F}$

(iii) For each of the following, state if the quantity is a scalar, a vector, or undefined. (You do not need to compute the quantity.) Be sure to justify your answer.

(a) **[2 points]** $\int_C f \mathrm{d}s$

(b) **[2 points]** $\int_C \mathbf{F} \cdot d\mathbf{r}$

Question 3 [20 points]

(i) Consider the following integral

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} \mathrm{d}y\mathrm{d}x$$

(a) **[5 points]** Sketch the region of integration.

(b) **[7 points]** Write down an equivalent integral, with the order of integration reversed. Make sure you clearly state the limits of integration. (Note, you do not need to evaluate the resulting integral.)

(ii) **[8 points]** Identify and sketch the surface given in spherical coordinates by $\{(\rho, \varphi, \theta) : \rho = 2\csc\varphi, \quad 0 < \varphi < \pi\}$.

Question 4 [20 points]

(i) **[10 points]** Set up, but do not evaluate, a double integral that represents the volume of the part of the cylinder $x^2 + y^2 = 1$ bounded above by the plane $z = 12 - x - y$ and below by $z = 0$. Make sure you clearly state the limits of integration and order of integration.

(ii) **[10 points]** Evaluate the integral

$$\iiint_D \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \mathrm{d}V,$$

where D is the region between the spheres of radius 1 and 2, centered at the origin.

Question 5 [20 points]

(i) **[5 points]** Is the vector field $\mathbf{F} = \langle 2x^3 + xy^2, 2y^3 - x^2y \rangle$ conservative? Be sure to justify your answer.

(ii) **[5 points]** State the Fundamental Theorem for Line Integrals. (No justification needed here - just a correct statement of the theorem.)

(iii) **[10 points]** Consider the line integral

$$\oint_C \mathbf{F} \cdot \mathbf{n} \, \mathrm{d}s,$$

where $\mathbf{F} = \langle 5y \sin x, x^2 + y^2 \rangle$, C is the circle of radius 1, centered at the origin, oriented counterclockwise, and $\mathbf{n}$ is the outward unit normal vector on the curve. Write down, but do not evaluate, an equivalent double integral.