Practice Final Exam – MA 225 A1 – Spring 2015

Problem	Possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

IMPORTANT: This practice exam does not cover all possible topics that may be on the exam. There may be questions on the actual exam that pertain to material not covered by this practice final. For the actual final, you are responsible for all material covered in this course.

Study suggestion: Do not look at this exam until you have "finished" studying. At that point, go somewhere no one will bother you for two hours, and take this exam as if it is the real exam. (Do not mess around with the internet, your phone, etc, while taking this practice exam.) Grade yourself, or better yet trade with a friend and grade each other. (If your friend cannot follow your work, then you have not explained yourself very well - this means your may not receive partial/full credit for something you think you understood.) Then go back and re-study the topics you wish to improve.

Question 1 [10 points] Evaluate

$$\iint_R \sin(9x^2 + 9y^2)\mathrm{d}A$$

where R is the region in the first quadrant bounded by the circle $x^2 + y^2 = 16$.

Question 2 [10 points]

(i) [**4 points**] Suppose $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is a vector and $\mathbf{F} = \mathbf{F}(x, y, z)$ is a vector field in $\mathbf{R}^3$. Is the following quantity meaningful?

$$\mathbf{b} \times (\nabla \cdot \mathbf{F})$$

If so, is it a scalar or a vector? (Make sure you justify your answer.)

(ii) [**5 points**] Are the following lines parallel?

$$\mathbf{r}_1(t) = \langle 1 + 2t, 3t, 4 - t \rangle, \qquad \mathbf{r}_2(t) = \langle -5 + 4t, 7 + 6t, 1 - 2t \rangle, \qquad -\infty < t < \infty$$

Question 3 [10 points] Find a parametric representation for the curve consisting of the part of the plane $z = x + 3$ that lies on the cylinder $x^2 + y^2 = 1$.

Question 4 [10 points]

Consider $\iint_S \mathbf{F}\cdot\mathbf{n}\mathrm{d}S$ where $\mathbf{F}(x,y,z) = \langle y, z-y, x\rangle$ and S is the surface of the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$. Write down, but do not evaluate, an equivalent triple integral. Be sure to specify the limits and order of integration in the triple integral.

Question 5 [10 points]

(i) Evaluate the integral

$$\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}}(x^2z+y^2z+z^3)\mathrm{d}z\mathrm{d}x\mathrm{d}y.$$

(ii) Determine the equation of the plane that contains the point $(0,1,0)$ and does not intersect the xz-plane.

Question 6 [10 points]

Determine whether or not the limit exists. If it exists, find its value. (Make sure to justify your answer.)

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{3x^2+2y^2}$$

Question 7 [10 points]

Sketch the solid whose volume is given by the integral

$$\int_0^1 \int_0^{1-x^2} (1-x)\mathrm{d}y\mathrm{d}x.$$

Question 8 [10 points]

Determine the area of the surface consisting of the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 9$.

Question 9 [10 points]

Show that the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ and the sphere $x^2 + y^2 + z^2 - 8x - 6y - 8z + 24 = 0$ are tangent to each other at the point $(1, 1, 2)$. (This means they have the same tangent plane at that point.)

Question 10 [10 points]

(i) Find f_z if

$$f(x, y, z) = e^{\sin(xyz)} + \frac{x^2y}{1+z^2}.$$

(ii) Find the directional derivative of $f(x, y, z) = x^2yz$ at the point $(1, 2, 3)$ in the direction of the vector $\langle 1, 0, 1 \rangle$.