Question 1 Using polar coordinates, we find
\[ \int \int_{R} \sin(9x^2 + 9y^2) \, dA = \int_{0}^{\pi/2} \int_{0}^{4} r \sin(9r^2) \, dr \, d\theta = \frac{\pi}{36} (1 - \cos(144)). \]

Question 2
(i) Since \( \nabla \cdot \mathbf{F} \) is a scalar, it does not make sense to take the cross product of this quantity and the vector \( \mathbf{b} \). Thus, it is not meaningful.

(ii) The direction vectors for the two lines are \( \mathbf{v}_1 = \langle 2, 3, -1 \rangle \) and \( \mathbf{v}_2 = \langle 4, 6, -2 \rangle \). These are scalar multiples of each other, \( \mathbf{v}_2 = 2\mathbf{v}_1 \), hence parallel. Since the direction vectors are parallel, the lines are parallel.

Question 3 The \( x \) and \( y \) components of the cylinder can be parameterized using \( x = \cos t, y = \sin t \), \( 0 \leq t \leq 2\pi \). Thus, \( z = \cos t + 3 \), and so we find
\[ \mathbf{r}(t) = \langle \cos t, \sin t, \cos t + 3 \rangle, \quad 0 \leq t \leq 2\pi. \]

Question 4 We will apply the divergence theorem. The plane forming the top of the tetrahedron is \( x + y + z = 1 \), and so
\[ \int \int \mathbf{F} \cdot n \, dS = \int \int \int \nabla \cdot \mathbf{F} \, dV = \int_{0}^{1} \int_{-x+1}^{1-x-y} \int_{0}^{-x} (-1) \, dz \, dy \, dx. \]

Question 5
(i) This is just a triple integral over the entire sphere of radius 3, and so we convert to spherical coordinates to find
\[ \int_{0}^{2\pi} \int_{0}^{\pi} \int_{0}^{3} \rho^5 \sin \varphi \cos \varphi \, d\rho \, d\varphi \, d\theta = 0. \]

(ii) Since the plane doesn’t intersect the \( xz \)-plane, it must be parallel to it, and hence have normal vector \( \langle 0, 1, 0 \rangle \). Thus, the plane is
\[ 0(x - 0) + 1(y - 1) + 0(z - 0) = 0 \quad \Rightarrow \quad y = 1. \]

Question 6 We check all lines of the form \( y = mx \):
\[ \lim_{x \to 0} \frac{x^2}{3x^2 + 2m^2x^2} = \frac{1}{3 + 2m^2} \]
Since this depends on \( m \), the limit does not exist.

Question 7 The region of integration in the \( xy \)-plane is the region enclosed by the parabola \( y = 1 - x^2 \) in the first quadrant. Thus, the integral represents the volume of the solid region that lies under the
plane \( z = 1 - x \) and above the previously described region in the \( xy \)-plane. Your drawing should reflect this.

**Question 8** We parameterize the surface by \( \mathbf{r}(x, y) = (x, y, x^2 + y^2) \) where \( x^2 + y^2 \leq 9 \). Thus,
\[
A(S) = \iint_R |\mathbf{r}_x \times \mathbf{r}_y| \, dA = \iint_D \sqrt{1 + 4(y^2 + z^2)} \, dA = \int_0^{2\pi} \int_0^3 r \sqrt{1 + 4r^2} \, dr \, d\theta = \frac{\pi}{6} [(37)^{3/2} - 1].
\]

**Question 9** We can think of each of these surfaces of level surfaces of functions: \( F(x, y, z) = k \). The normal vector of the tangent plane of such a surface at a point \((x_0, y_0, z_0)\) is \( \nabla F(x_0, y_0, z_0) \). Thus, the normal vector for the tangent plane to the ellipsoid is \( \mathbf{n}_1 = (6, 4, 4) \) and the normal vector for the tangent plane to the sphere is \( \mathbf{n}_2 = -(6, 4, 4) \). These vectors are parallel, because they are scalar multiples of each other, and both tangent planes contain the point \((1, 1, 2)\). Since any two planes that are parallel and contain a common point must be the same plane, the ellipsoid and sphere are tangent at that point.

**Question 10**

(i) We have
\[
f_z = xy \cos(xyz) e^{\sin(xyz)} - \frac{2x^2yz}{(1 + z^2)^2}.
\]

(ii) Since \( \mathbf{u} = (1, 0, 1)/\sqrt{2} \) and \( \nabla f(1, 2, 3) = (12, 3, 2) \), we have
\[
D_{\mathbf{u}} f = (1, 0, 1)/\sqrt{2} \cdot (12, 3, 2) = \frac{14}{\sqrt{2}}.
\]