MA 226 Differential Equations
Summer II, 2003 Exam #1

INSTRUCTIONS: Please read all problems carefully and write your solutions on the paper provided. Show all your work - no credit will be given unless all work is shown. Don't forget to put your name on your solution sheets.

1. Find the general solution to the following differential equations using any analytic technique you wish:

   a) (8 points)

$$\frac{dy}{dt} = \frac{t}{t^2 y + y}$$

   b) (8 points)

$$\frac{dy}{dt} - 3y = 4e^{2t}$$

   c) (8 points)

$$\frac{dy}{dt} = 2ty + 7e^{t^2}$$

2. Suppose you are given the equation

$$\frac{dy}{dt} = f(t, y)$$

   and a step size Δt.

   a) (5 points) If you know the point (t_k, y_k), please state the formulas given by Euler's method for the point (t_{k+1}, y_{k+1}).

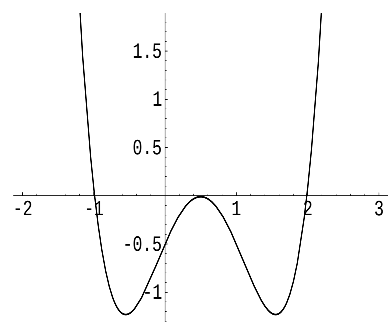
b) (3 points) Use Euler's method to approximate the solution to the initial value problem

$$\frac{dy}{dt} = 3t^2 y, \quad y(1) = 2$$

on the interval $1 \le t \le \frac{3}{2}$ with step size $\Delta t = 0.5$.

c) (5 points) Please draw a picture illustrating how Euler's method works and explain where the formulas you wrote down in part a) come from.

3. Suppose you are given the differential equation $\frac{dy}{dt} = f(y)$ and graph of the function f(y) shown below:



a) (4 points) What are the equilibrium values?

b) (8 points) For what values of y are solutions increasing? For what values of y are solutions decreasing?

c) (10 points) Please sketch the slope field and the graphs of solutions for $y(0) = -2$, $y(0) = 1$, and $y(0) = 2$.

d) (8 points) Please draw the phase line and classify the equilibria.

4. Suppose the fish population in a particular lake can be modeled by the logistic population model with growth rate $k = 4$ and carrying capacity $N = 1$. Also suppose that C fish are caught each year. If t represents time in years the corresponding equation is

$$\frac{dy}{dt} = 4y(1 - y) - C$$

a) (10 points) Please locate the bifurcation value and draw phase lines for values of the parameter slightly smaller than, slightly larger than, and at the bifurcation value.

b) (8 points) Please sketch the graph of the right hand side of the equation, $f_C(y)$, for different values of $C > 0$. Explain how increasing the value of C changes the graph and causes the bifurcation.

c) (2 points) What happens to the fish population if the value of C is larger than the bifurcation value?

5. Consider the following initial value problem:

$$\frac{dy}{dt} = y^{1/2}, \quad y(0) = 0$$

a) (8 points) Please find two different solutions to the initial value problem.

b) (5 points) Why doesn't this contradict the Existence and Uniqueness Theorem?

6. **Extra Credit** This problem is worth 5 points - no partial credit will be given.

   Please explain why the function $y(t)$ graphed below CANNOT be a solution to an autonomous, first-order differential equation.