**MA 122**
**Summer II 2017**
**Practice Final Exam**

**Name (Print):** ______________________________

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This exam contains 10 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You can **NOT** use your books or any calculators on this exam, but you can take one page of notes.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- The last problem is for **extra credits**. It is **much more difficult**, so only attempt it if you have completed the other problems as best you can.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

| Problem | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 15 | |
| 4 | 10 | |
| 5 | 15 | |
| 6 | 15 | |
| 7 | 25 | |
| 8 | 10 | |
| Total: | 110 | |

1. (10 points) Find $f^{(n)}(x)$, where $f(x) = e^{2x}$. [Suggested Practice: Chap 2.1 - 31, 34, 36.]

2. (10 points) If $y$ is defined implicitly by $y + e^{y^2} - x^2 = C$, show that $y$ is the general solution to the differential equation $(1+2ye^{y^2})y' = 2x$. Then, find a particular solution satisfying $y(1) = 0$. [Suggested Practice: Chap 1.1 - 31, 34, 36.]

3. (15 points) There are 3 points $(1,1)$, $(2,\frac{1}{2})$ and $(4,\frac{1}{4})$, obtained from $f(x)=\frac{1}{x}$. [Suggested Practice: Chap D.1 - 13, 37.]

   (a) (5 points) Find the interpolating polynomial for these data using Divided Difference Table.

   (b) (5 points) Find the expression of piecewise linear approximation for these data.

   (c) (5 points) Graph the functions obtained from (a), (b) and $f(x)=\frac{1}{x}$ in one figure in the domain $1\le x\le 4$. Use the results from (a) and (b) to approximate the function value at $x=3$. Which approximation method is better? Explain why.

4. (10 points) Given $(\sin x)' = \cos x$ and $(\cos x)' = -\sin x$, find the 3rd-degree Taylor polynomial at 0 for $f(x) = 2\sin x \cos x$ and use it to approximate $f(\frac{\pi}{6})$. Is it a good approximation? [Suggested Practice: Chap 2.1 - 13, 14.]

5. (15 points) [Suggested Practice: Chap 2.3 - 33, 40, 49; Chap 2.4 - 26, 28.]

   (a) (5 points) Find the Taylor series at 0 for $\frac{1}{9+x^2}$ and interval of convergence.

   (b) (5 points) Find the Taylor series at 0 for $\int \frac{1}{9+x^2}\, dx$ and interval of convergence.

   (c) (5 points) Approximate the integral $\int_0^1 \frac{1}{9+x^2}\, dx$ to within $\pm 0.0005$.

6. (15 points) Given a differential equation $xy' + y = 2x$. [Suggested Practice: Chap 1.3 - 14, 15.]

   (a) (10 points) Find the general solution.

   (b) (5 points) Find a particular solution satisfying $y(1) = 1$.

7. (25 points) A rumor spreads through a population of 1000 people at a rate proportional to the product of the number who have heard it and the number who have not heard it. Suppose 5 people initiated a rumor and 10 people had heard it after 1 day. [Suggested Practice: Chap 1.2 - 40, 45, 68, 69.]

   (a) (5 points) Translate the problem in the language of differential equation with initial condition? What is the model you are using? (In this part, you don't need to determine the rate.)

   (b) (10 points) Solve the differential equation in (a) with initial condition. Determine the spread rate based on the information in the problem.

(c) (5 points) What is the equilibrium of the system? Is it dynamically stable? Explain why.

(d) (5 points) How many people will have heard the rumor after 7 days? How long will it take for 850 people to hear the rumor? (You don't need to calculate the final number, but you need to provide the final expression of the answer.)

8. (10 points) (*Extra Credits*) Draw the slope field of $y' = (x-1)(y-1)$. Based on the slope field, draw the particular solution satisfying $y(2) = 2$.