MA 122
Summer II 2017
Practice Midterm Exam Keys

Name (Print): ____________________

This exam contains 14 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You can **NOT** use your books or any calculators on this exam, but you can take one page of notes.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- The last problem is for **extra credits**. It is **much more difficult**, so only attempt it if you have completed the other problems as best you can.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	5	
5	15	
6	25	
7	25	
8	10	
Total:	110	

1. (10 points) Find $f_y(2,2)$ and $f_x(2,1)$, where $f(x,y) = x^4 \ln y^2$.

Answer:

Note that $f(x,y) = x^4 \ln y^2 = 2x^4 \ln y$.

$f_y(x,y) = 2x^4/y$, so $f_y(2,2) = 16$.

$f_x(x,y) = 8x^3 \ln y$, so $f_y(2,1) = 0$.

2. (10 points) Find $\int f(x,y)\ \mathrm{d}x$ and $\int f(x,y)\ \mathrm{d}y$, where $f(x,y) = \frac{\ln x}{xy}$.

Answer:

choose $u = \ln x$, then $\mathrm{d}u = \frac{1}{x}\mathrm{d}x$.

$\int f(x,y)\ \mathrm{d}x = \int u/y\ \mathrm{d}u = \frac{1}{2}u^2/y + C(y) = \frac{1}{2}\ln^2 x/y + C(y)$.

$\int f(x,y)\ \mathrm{d}y = \frac{\ln x}{x}\int 1/y\ \mathrm{d}y = \frac{\ln x}{x}\ln|y| + C(x)$.

3. (10 points) Method of Least Squares.

   (a) (8 points) Find the least squares line referring to $n = 3$ data points $(x_1, y_1) = (0, -1)$, $(x_2, y_2) = (1, 0)$, and $(x_3, y_3) = (2, 0)$.

   Answer:

   Use the least squares formula, you will get $a = \frac{1}{2}$, $b = -\frac{5}{6}$. The line of equation is $y = \frac{1}{2}x - \frac{5}{6}$.

   (b) (2 points) Use the line in part (a) to estimate y when $x = 5$.

   Answer:

   By plugging $x = 5$ into $y = \frac{1}{2}x - \frac{5}{6}$, we will have $y = \frac{5}{3}$.

4. (5 points) Consider the function $f(x) = \cos 2x + \sin x$.

   (a) (3 points) Find $f(\frac{\pi}{6})$.

   Answer:

   $f(\frac{\pi}{6}) = \cos\frac{\pi}{3} + \sin\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2} = 1.$

   (b) (2 points) Is $f(x)$ a periodic function? Explain why.

   Answer:

   Yes, because $f(x + 2\pi) = \cos(2x + 4\pi) + \sin(x + 2\pi) = \cos 2x + \sin x = f(x)$.

5. (15 points) Consider the function $f(x) = \tan x$.

   (a) (4 points) Find the domain, range and period of $f(x)$.

   Answer:

   Domain is $x \neq \frac{\pi}{2} + n\pi$, where n is any integer; Range is $(-\infty, +\infty)$; Period is π.

   (b) (4 points) Find $f'(x)$.

   Answer:

   $f'(x) = \frac{1}{\cos^2 x}$.

(c) (3 points) Graph $y = f(x)$ on the interval $(-\pi/2, \pi/2)$.

Answer:

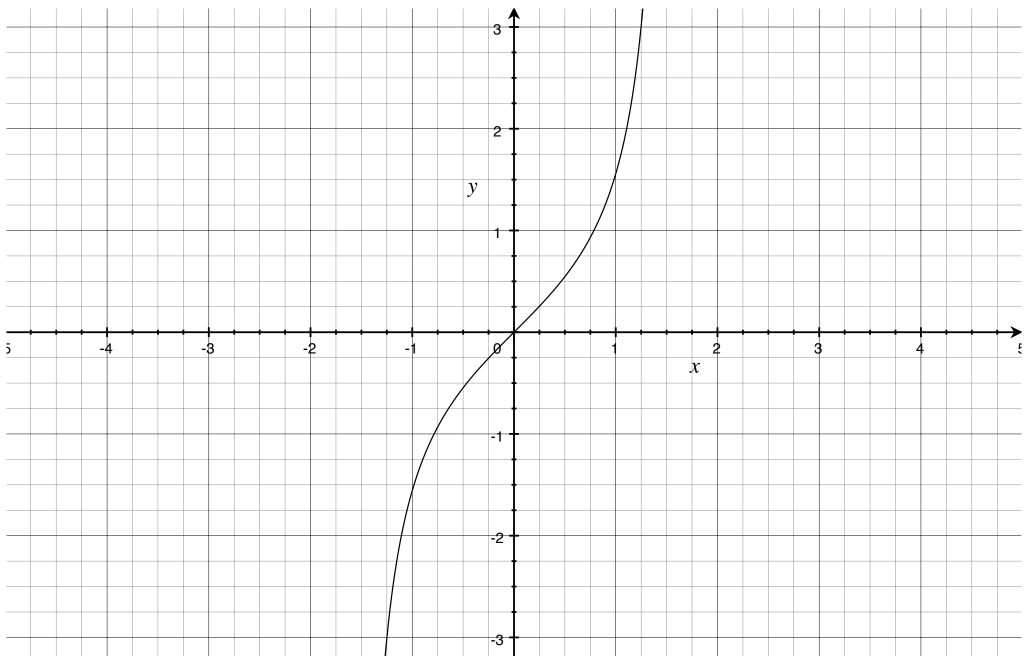


Figure 1: $y = \tan x,\ x \in (-\pi/2, \pi/2)$

(d) (4 points) Find $\int f(x)\ \mathrm{d}x$.

Answer:

Choose $u = \cos x$, then $\mathrm{d}u = -\sin x \mathrm{d}x$

$\int f(x)\ \mathrm{d}x = \int -\frac{1}{u}\mathrm{d}u = -\ln|u| + C = -\ln|\cos x| + C.$

6. (25 points) Consider the function $f(x, y) = x^2 - x^2y^2 - y^2$.

   (a) (5 points) Find the equations of the cross sections of the surface $z = f(x, y)$ produced by cutting it with planes $x = 0$ and $y = 0$ and graph the curves. Can you guess whether point (0, 0) is a local minima point, local maxima point or a saddle point based on these two cross sections? Explain why.

   Answer:

   $z = -y^2$ and $z = x^2$, (Graphs here are omitted.)

   Saddle Point, because it is minima point in one plane and maxima point in the other plane.

(b) (10 points) Use the Second-Derivative Test for Local Extrema to find local extrema of $z = f(x, y)$. Does it coincide with the result you got from part (a)?

Answer:

Since $f_x(x, y) = 2x - 2xy^2$ and $f_y(x, y) = -2yx^2 - 2y$, we have only one critical point (0, 0).

Since $f_{xx}(0, 0) = 2$, $f_{xy}(0, 0) = 0$ and $f_{yy}(0, 0) = -2$, we can conclude that (0, 0) is a saddle point based on Second-Derivative Test for Local Extrema, as expected in part (a).

(c) (10 points) Minimize $f(x, y)$, subject to $x^2 + y^2 = 4$.

Answer:

$F(x, y, \lambda) = x^2 - x^2y^2 - y^2 + \lambda(x^2 + y^2 - 4)$.

Since $F_x(x, y, \lambda) = 2x - 2xy^2 + 2x\lambda$, $F_y(x, y, \lambda) = -2yx^2 - 2y + 2y\lambda$ and $F_\lambda(x, y, \lambda) = x^2 + y^2 - 4$, we have critical points $(0, \pm 2)$, $(\pm 2, 0)$ and $(\pm 1, \pm\sqrt{3})$.

$f(x, y)$ on these critical points are equal to-4, 4 and -5, so the minimal value for $f(x, y)$ is -5.

7. (25 points) Double Integrals over regular regions.

   (a) (10 points) Evaluate $\int_0^1 \int_{-2x}^{x^2} xy \; \mathrm{d}y \; \mathrm{d}x$.

   Answer:

   $\int_0^1 \int_{-2x}^{x^2} xy \; \mathrm{d}y \; \mathrm{d}x = \int_0^1 \frac{1}{2}xy^2|_{y=-2x}^{y=x^2} \; \mathrm{d}x = \int_0^1 \frac{1}{2}x^5 - 2x^3 \; \mathrm{d}x = (\frac{1}{12}x^6 - \frac{1}{2}x^4)|_{x=0}^{x=1} = -\frac{5}{12}$.

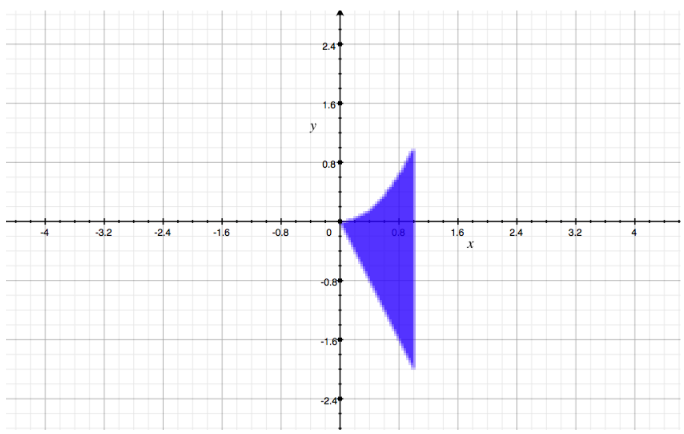
(b) (5 points) Graph the region of integration and describe it as a regular x region and a regular y region.

Answer:

$R = \{(x,y)|0 \leq x \leq 1, \ -2x \leq y \leq x^2\} = \{(x,y)| -2 \leq y \leq 1, \ h(y) \leq x \leq 1\}$, where

$$h(y) = \begin{cases} -y/2 & -2 \leq y \leq 0 \\ \sqrt{y} & 0 \leq y \leq 1 \end{cases}$$

The region is shown as follows.



(c) (10 points) Reverse the order of integration and evaluate the integral with the order reversed. Do you get the same result as part (a) ?

Answer:

$\int_{-2}^{1}\int_{h(y)}^{1} xy \text{ d}x \text{ d}y = \int_{-2}^{0}\int_{-y/2}^{1} xy \text{ d}x \text{ d}y + \int_{0}^{1}\int_{\sqrt{y}}^{1} xy \text{ d}x \text{ d}y.$

$\int_{-2}^{0}\int_{-y/2}^{1} xy \text{ d}x \text{ d}y = \int_{-2}^{0} \frac{1}{2}x^2y|_{x=-y/2}^{x=1} \text{ d}y = \int_{-2}^{0} \frac{1}{2}y - \frac{1}{8}y^3 \text{ d}y = (\frac{1}{4}y^2 - \frac{1}{32}y^4)_{y=-2}^{y=0} = -\frac{1}{2},$

$\int_{0}^{1}\int_{\sqrt{y}}^{1} xy \text{ d}x \text{ d}y = \int_{0}^{1} \frac{1}{2}x^2y|_{x=\sqrt{y}}^{x=1} \text{ d}y = \int_{0}^{1} \frac{1}{2}y - \frac{1}{2}y^2 \text{ d}y = (\frac{1}{4}y^2 - \frac{1}{6}y^3)|_{y=0}^{y=1} = \frac{1}{12},$

Therefore, $\int_{-2}^{1}\int_{h(y)}^{1} xy \text{ d}x \text{ d}y = -\frac{1}{2} + \frac{1}{12} = -\frac{5}{12}.$

The same result as part (a)!

8. (10 points) (*Extra Credits*) Evaluate $\int \frac{(\sec x)^2+\sec x \tan x}{\sec x+\tan x}\,\mathrm{d}x$.

Answer:

Choose $u = \sec x + \tan x$, then $\mathrm{d}u = ((\sec x)^2 + \sec x \tan x)\mathrm{d}x$.

$\int \frac{(\sec x)^2+\sec x \tan x}{\sec x+\tan x}\,\mathrm{d}x = \int 1/u\mathrm{d}u = \ln|u| + C = \ln|\sec x + \tan x| + C$.