Quiz 1

NAME:

Question 1.(6 POINTS.) FIND THE DOMAIN, DERIVATIVE AND LOCAL EXTREMA (MINIMUM OR MAXIMUM) OF $f(x) = \ln x + \frac{1}{\sqrt[3]{x}}$.

DOMAIN: $(0, +\infty)$

DERIVATIVE: $f'(x) = \frac{1}{x} - \frac{1}{3}x^{-\frac{4}{3}} = x^{-1}(1 - \frac{1}{3}x^{-\frac{1}{3}})$

LOCAL EXTREMA:

ONLY $x = \frac{1}{27}$ MAKES $f'(x) = 0$.

WHEN $0 < x < \frac{1}{27}$, $f'(x) < 0$; WHEN $x > \frac{1}{27}$, $f'(x) > 0$.

THUS, $(\frac{1}{27}, f(\frac{1}{27}))$ IS LOCAL MINIMUM.

Question 2.(4 POINTS.) EVALUATE $\int_{-1}^{1} x\mathrm{e}^{-x^2}\,\mathrm{d}x$.

METHOD 1:

DENOTE $f(x) = x\mathrm{e}^{-x^2}$. NOTE THAT $f(x) = -f(-x)$, SO $\int_{-1}^{1} f(x)\,\mathrm{d}x = 0$.

METHOD 2:

LET $u = x^2$, THEN $\int x\mathrm{e}^{-x^2}\,\mathrm{d}x = \frac{1}{2}\int \mathrm{e}^{-u}\,\mathrm{d}u = -\frac{1}{2}e^{-u} + C = -\frac{1}{2}e^{-x^2} + C$.

THEREFORE, $\int_{-1}^{1} x\mathrm{e}^{-x^2}\,\mathrm{d}x = -\frac{1}{2}e^{-1^2} - (-\frac{1}{2}e^{-(-1)^2}) = 0$.