Quiz 5

NAME:

Question 1.(5 points.) Find the third-degree Taylor polynomial at 0 for $f(x) = \ln(1+2x)$.

Step 1 - Find derivatives:

$f'(x) = \frac{2}{1+2x}$, $f''(x) = -\frac{4}{(1+2x)^2}$ and $f'''(x) = \frac{16}{(1+2x)^3}$.

Step 2 - Find derivatives at 0:

$f(0) = 0,\ f'(0) = 2,\ f''(x) = -4$ and $f'''(x) = 16$.

Step 3 - Find coefficients of Taylor polynomial:

$a_0 = 0,\ a_1 = 2,\ a_2 = -2$ and $a_3 = \frac{8}{3}$.

Step 4 - Write down the Taylor polynomial:

$p_3(x) = 2x - 2x^2 + \frac{8}{3}x^3$.

Question 2.(5 points.) Given that the interval of convergence of the Taylor series $\ln(1-x) = -\sum_{n=1}^{\infty} \frac{1}{n}x^n$ at 0 is $-1 < x < 1$, find the Taylor Series of function $\ln(2-2x+x^2)$ at 1 and interval of convergence.

$\ln(2-2x+x^2) = \ln(1+(x-1)^2) = \sum_{n=1}^{\infty} \frac{1}{n}(-(x-1)^2)^n = \sum_{n=1}^{\infty} \frac{1}{n}(-1)^n(x-1)^{2n}$

where the interval of convergence is $-1 < -(x-1)^2 < 1$, which is $0 < x < 2$.