Quiz 6

NAME:

Question 1.(4 POINTS.) FIND THE GENERAL SOLUTION OF $y' = (2x + 4)(y - 3)$ USING THE METHOD OF "SEPARATION OF VARIABLES".

$$\frac{1}{y - 3}y' = 2x + 4$$

THEN, BY THE THEOREM OF SEPARATION OF VARIABLES,

$$\int \frac{1}{y - 3} \, dy = \int 2x + 4 \, dx$$

$$\ln |y - 3| = x^2 + 4x + C$$

$$y = Ce^{x^2 + 4x} + 3$$

Question 2.(4 POINTS.) FIND THE GENERAL SOLUTION OF $y' = (2x+4)(y-3)$ USING THE METHOD FOR SOLVING "FIRST-ORDER LINEAR DIFFERENTIAL EQUATIONS".

STANDARD FORM:

$$y' - (2x + 4)y = -3(2x + 4)$$

INTEGRATING FACTOR: $I(x) = e^{\int -(2x+4) \, dx} = e^{-x^2 - 4x}$

MULTIPLY BOTH SIDES BY $I(x)$ AND INTEGRATE BOTH SIDES:

$$e^{-x^2 - 4x}y = -3 \int e^{-x^2 - 4x}(2x + 4) \, dx = 3e^{-x^2 - 4x} + C$$

$$y = -3 \int e^{-x^2 - 4x}(2x + 4) \, dx = 3 + Ce^{x^2 + 4x}$$

Question 3.(2 POINTS.) USING THE RESULT FOR QUESTION 1 OR QUESTION 2, FIND THE PARTICULAR SOLUTION OF $y' = (2x + 4)(y - 3)$ SATISFYING $y(0) = 2$.

PLUGGING $y(0) = 2$ INTO THE GENERAL SOLUTION, YOU'LL HAVE

$$2 = 3 + Ce^0$$

THIS IMPLIES $C = -1$, SO PARTICULAR SOLUTION IS $y = 3 - e^{x^2 + 4x}$