

Lecture on July 9th, 2018
First Step Analysis and Two-State Model

1 General First Step Analysis - See Chap 3.4.2

- Idea: analyze the first step and then use LTP + Markov property(M.P.)
- Example: Let $\{X_n\}$ be a MC with finite states: $0, \dots, r-1$ to be transient and r, \dots, N to be absorbing.

Question:

- (1) Distribution of states over absorption?
- (2) Mean time of absorption (More generally, $\mathbb{E}[\sum_{n=0}^{T-1} g(X_n) | X_0 = i]$)?

2 Two-State Model - See Chap 3.5.1

- Let $\{X_n\}$ be a MC with

$$P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$$

where $0 < a, b < 1$.

- If $1-a = b$, X_n 's are i.i.d.
- If $1-a \neq b$, distribution of X_{n+1} depends on the value of X_n .

– Claim (Check by induction):

$$P^{(n)} = \frac{1}{a+b} \begin{pmatrix} b & a \\ b & a \end{pmatrix} + \frac{(1-a-b)^n}{a+b} \begin{pmatrix} a & -a \\ -b & b \end{pmatrix}.$$

– As $n \rightarrow \infty$,

$$\lim_{n \rightarrow \infty} P^{(n)} = \frac{1}{a+b} \begin{pmatrix} b & a \\ b & a \end{pmatrix}.$$

– The long-time limiting distribution doesn't depend on the initial distribution!