Lecture on July 10th, 2018
Some Special Markov Chains

1 Markov Chains Defined by Random Variables

- Ex: independent r.v. $X_n = \xi_n$, where ξ_n's are i.i.d. In this case, the transition matrix doesn't depends on initial states.
- Ex: Successive Maxima. $X_n = \max\{\xi_1, \xi_2, \cdots, \xi_n\}$ for $n \geq 1$ and $X_0 = 0$. Check this is a M.C.
- Ex: Partial Sums. $X_n = \sum_{i=1}^{n} \xi_i$, for $n \geq 1$ and $X_0 = 0$. Check that this is a M.C.

2 Gambler's Ruin Problem

- Assume 0 and N are two absorbing states. Let u_i be the probability of reaching state 0 starting at fortune i.
- By first step analysis,

$$\begin{cases} u_0 = 1, u_N = 0, \\ u_i = pu_{i+1} + qu_{i-1}, \quad i = 1, 2, \cdots, N-1 \end{cases}$$

- After some calculations, ...

$$u_i = \begin{cases} \frac{N-i}{N}, & \text{if } p = q \\ \frac{(\frac{q}{p})^i - (\frac{q}{p})^N}{1-(\frac{q}{p})^N}. & \text{if } p \neq q \end{cases}$$

- In the limiting case as $N \to \infty$,

$$u_i = \begin{cases} 1 & \text{if } p \leq q \\ (\frac{q}{p})^i. & \text{if } p > q \end{cases}$$

3 Success Runs

A special case: Consider successive trials with 2 outcomes: S, F. X_n represents the length of success runs at current time n. Then we have only two cases to update X_n's.

- If next trial success (with probability q): $X_{n+1} = X_n + 1$
- If next trial fails (with probability p): $X_{n+1} = 0$