Lecture on July 16th, 2018
Periodicity and Recurrence

1 Last Example on communication class (See 4.3.1)

- One dimensional random walk with two absorbing boundaries: communication classes are $\{0\}$, $\{N\}$ and $\{1, 2, \cdots, N-1\}$.

2 Periodicity (See 4.3.2)

- Definitions and examples of "period" of state i, $d(i)$ and "aperiodic".
- 3 properties of "period". Periodicity is a class property.
- For finite state Markov chain, regular is equivalent to irreducible + aperiodic.

3 Recurrence (See 4.3.3)

- Definitions and examples of "recurrent" and "transient".
- Distribution and mean of number of times returns to state i.
- Theorem: a state i is recurrent iff $\sum_{n=1}^{\infty} p_{ii}^{(n)} < \infty$.
- Recurrence is a class property.
- Example: One dimensional random walk on $\mathbb{Z}$. State 0 is recurrent if $p = q = \frac{1}{2}$, otherwise, state 0 is transient.