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Lecture on July 2nd, 2019
Conditional Probability and Conditional Expectations II

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1 Random Sums - See Chap 2.3

- Definition of random sum.

- Definition and properties of conditional distribution function and density function.

- Moments (mean and variance) of random sum.

- Distribution of random sum.

- Example: $\xi_i \sim Exp(\lambda)$, $N \sim Geo(\beta)$, then random sum $X \sim Exp(\lambda\beta)$.

2 Conditional Probability and Expectations (Continuous Case) - See Chap 2.4

- Definitions and properties of conditional probability density function, conditional distribution function, conditional expectation.

- Example: If $X, Y \sim f_{X,Y}(x,y) = \frac{1}{y}\exp^{-x/y-y}$ for $x, y > 0$, then $f_{X|Y}(x|y) = \frac{1}{y}\exp^{-x/y}$, for $x, y > 0$.

3 Martingales - See Chap 2.5

- Definition of stochastic process, discrete-time and continuous time stochastic process.

- Definition and properties of discrete-time martingales.

- Martingales have constant mean.

- Martingale property: $\mathbb{E}[X_{n+1}|X_0, X_1, \cdots, X_n] = X_n$ or equivalently $\mathbb{E}[X_m|X_0, X_1, \cdots, X_n] = X_n$, for any $m \geq n$.

- Markov inequality