1 Martingales - See Chap 2.5

- Markov inequality and Maximal inequality for martingales.
- Example: Let \( \{X_n, n \geq 0\} \) be a gambler’s fortune. \( X_0 = 1 \) and he is facing a series of independent fair games with the following rules:

\[
X_{n+1} = \begin{cases} 
(1 + p)X_n & \text{with probability } 1/2, \\
(1 - p)X_n & \text{with probability } 1/2.
\end{cases}
\]

Check that \( \{X_n, n \geq 0\} \) is a non-negative martingale. Using maximal inequality, we can see that the probability to double money \( \leq 1/2 \).

2 Markov Chains (MC) - See Chap 3.1-3.2

- Definition of Markov processes in general.
- Definition of discrete-time Markov Chains, stationary Markov Chain and Markov property.
- Notations about one-step transition probability and Markov matrix (transition probability matrix).
- Properties of Markov matrix \( P \).
- Theorem: A discrete-time stationary MC is completely specified by its initial distribution and transition probability matrix.
- n-step transition probability matrix \( P^{(n)} \) and \( P^{(n)} = P^n \).