Lecture on July 3rd, 2019
Martingales and Introduction to Markov Chains

1 Martingales - See Chap 2.5

- Markov inequality and Maximal inequality for martingales.
- Example: Let $\{X_n, n \geq 0\}$ be a gambler's fortune. $X_0 = 1$ and he is facing a series of independent fair games with the following rules:

$$X_{n+1} = \begin{cases} (1+p)X_n & \text{with probability } 1/2, \\ (1-p)X_n & \text{with probability } 1/2. \end{cases}$$

  Check that $\{X_n, n \geq 0\}$ is a non-negative martingale. Using maximal inequality, we can see that the probability to double money $\leq 1/2$.

2 Markov Chains (MC) - See Chap 3.1-3.2

- Definition of Markov processes in general.
- Definition of discrete-time Markov Chains, stationary Markov Chain and Markov property.
- Notations about one-step transition probability and Markov matrix (transition probability matrix).
- Properties of Markov matrix P.
- Theorem: A discrete-time stationary MC is completely specified by its initial distribution and transition probability matrix.
- n-step transition probability matrix $P^{(n)}$ and $P^{(n)} = P^n$.