Lecture on July 11th, 2019
Some Special Markov Chains II

1 Gambler's Ruin Problem (See 3.5)

- Assume 0 and N are two absorbing states. Let u_i be the probability of reaching state 0 starting at fortune i.
- By first step analysis,

$$\begin{cases} u_0 = 1, u_N = 0, \\ u_i = pu_{i+1} + qu_{i-1}, \quad i = 1, 2, \cdots, N-1 \end{cases}$$

- After some calculations, ...

$$u_i = \begin{cases} \frac{N-i}{N}, & \text{if } p = q \\ \frac{(\frac{q}{p})^i - (\frac{q}{p})^N}{1-(\frac{q}{p})^N}. & \text{if } p \neq q \end{cases}$$

- In the limiting case as $N \to \infty$,

$$u_i = \begin{cases} 1 & \text{if } p \leq q \\ (\frac{q}{p})^i. & \text{if } p > q \end{cases}$$

2 Success Runs (See 3.5)

A special case: Consider successive trials with 2 outcomes: S, F. X_n represents the length of success runs at current time n. Then we have only two cases to update X_n's.

- If next trial success (with probability q): $X_{n+1} = X_n + 1$
- If next trial fails (with probability p): $X_{n+1} = 0$

3 Renewal Process (See 3.5.4)

- Suppose the lifetime for each light bulb is i.i.d. positive integer valued r.v. Each light bulb is replaced by a new one once it burns out. Let X_n to be the age of light bulb currently in service at time n.
- Find the transition probabilities.

4 Branching Process (See 3.8)

- An individual at the end of lifetime has random number of offsprings ξ taking non-negative integer values. All offsprings are independent of each other and follow the same distribution to propagate species. Let X_n be the size of population at time $n \geq 1$ and $X_0 = 1$.
- Find the mean and variance of X_n using conclusions in partial sums.