Mathematical Models in the Life Sciences

Assignment 3

Due: 10:00 AM **February 11**, 2008

UPDATED: Feb 2, 08. Typo in Problem 1 found by PJ Liu. Eqns (1) and (2) corrected.

You are free to use any books or notes as you work these problems. You are encouraged to work with other members of the class, but not to the point where you simply copy anothers work. Also feel free to ask me any questions about these problems. Your work should be neatly and carefully written, and be sure to show all of your work. Solutions to the collected problems will be available following the due date.

- Reading: Edelstein-Keshet — Chapter 2, excluding 2.3 and 2.10.
- Optional Reading [see course webpage]:
  R. M. May, *Simple mathematical models with very complicated dynamics*, Nature, 1976.
- Course webpage: http://math.bu.edu/people/mak/MA565
- Problems:

  1. **The Love Affairs model**
     In class, we discussed the *Love Affairs* model. We proposed that — at time n — the relationship between Romeo (R_n) and Juliet (J_n) evolved according to the system of linear difference equations:

     $$R_{n+1} = a_R R_n + p_R J_n \tag{1}$$
     $$J_{n+1} = a_J J_n + p_J R_n \tag{2}$$

     The parameters $\{a_R, a_J\}$ describe the impact of each individuals own feelings on himself or herself, and the parameters $\{p_R, p_J\}$ describe how each individuals feelings affect the other's feelings.

     Assume that $p_R, p_J \gg 1$ and $a_R, a_J \ll 1$ and answer the questions below.

     (a) Define in words the implications of these assumptions.

(b) Set $p_R, p_J = 10$ and $a_R, a_J = 0$, and compute the eigenvalues and corresponding eigenvectors for this system of equations.

(c) Consider the initial conditions $(R_0, J_0) = (+\epsilon, -\epsilon)$, where ϵ is very small. What happens over time to their feelings? (HINT: Try expressing the initial condition in terms of the eigenvectors!)

2. Use the cobweb technique to determine the fixed points and stabilities of the following difference equations. Please hand in your plot of the function and some iterates of the cobweb procedure. Also please label the fixed points and indicate their stabilities. Approximate! Hand-drawn sketches are fine.

   (a) $x_{n+1} = -x_n^3$

   (b) $x_{n+1} = 2x_n - x_n^2$

3. **Logistic Map Computer Exercises**

   In class, we've considered the logistic map: $x_{n+1} = rx_n(1-x_n)$. In this exercise, you will use the computer to study the dynamics of the logistic map. The computer is *essential* when the map becomes too complicated for analytic solutions and hand-drawn cobweb procedures. To start:

   (a) Examine the plot on the last page of this assignment; it's the logistic map with $r = 3.75$. Try to determine the stability of the fixed points by following the cobweb procedure. Do this by hand. What happens?

   (b) Go to the course webpage and click on the link to the *Nonlinear Web*. There you will find a JAVA applet that computes the cobweb procedure for the logistic map. Use the slider on the right of the figure to adjust the parameter r (note that as you adjust the slider, the equation at the bottom of the plot changes and the blue curve moves up and down.) Set $r = 3.75$ and click anywhere on the figure to chose the initial point to iterate — the first step in the cobweb procedure will occur. Click *Iterate* at the bottom of the figure to continue the cobweb procedure; each click of *Iterate* will cause another step in the cobweb procedure. What do you find?

   (c) Set $r = 3.25$ and iterate in the JAVA applet. Now what do you find?

4. Edelstein-Keshet — Chapter 2: 1.b, 1.c, 2.b, 2.c, 2.d, 3.

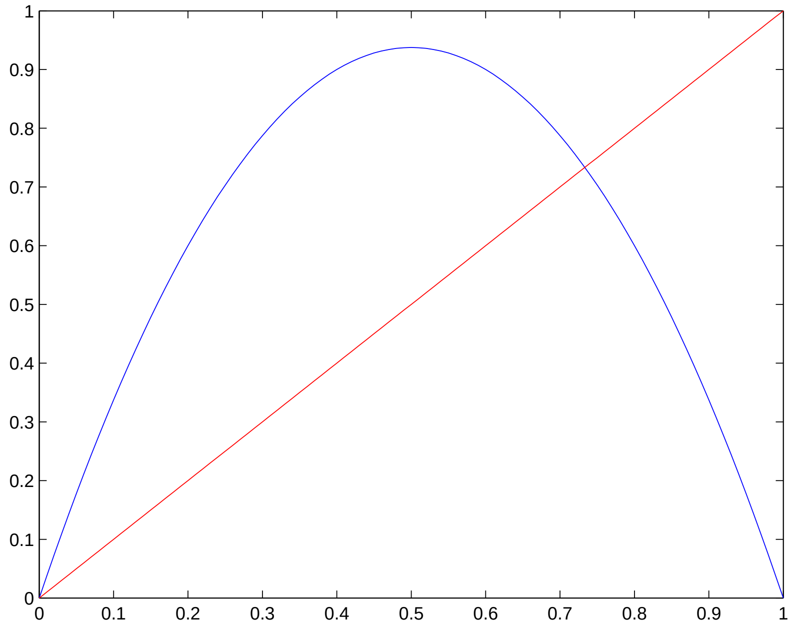

Figure 1: Logistic map: $x_{n+1} = rx_n(1 - x_n)$ with $r = 3.75$. Blue: the right hand side of this equation. Red: the diagonal line. Choose an initial condition and cobweb.