Mathematical Models in the Life Sciences

Assignment 4

Due: 10:00 AM February 19, 2008

You are free to use any books or notes as you work these problems. You are encouraged to work with other members of the class, but not to the point where you simply copy another's work. Also feel free to ask me any questions about these problems. Your work should be neatly and carefully written, and be sure to show all of your work. Solutions to the collected problems will be available following the due date.

- READING: Edelstein-Keshet — Chapter 1, 1.7 and 1.8. Chapter 2, excluding 2.3 and 2.10.
- OPTIONAL READING: Edelstein-Keshet — Chapter 3 for more examples of nonlinear difference equations.
- REMINDER: Test! In class on Friday, Feb 22.
- PROBLEMS:

1. **Chapter 1**: 8a, 8b.
2. **Chapter 2**: 11, 15.
   Note: For problem (15), find the fixed points and check their stabilities analytically. Then construct a cobweb diagram for this system and use this diagram to determine how the fixed points change as the parameter a increases. Do you observe a bifurcation? (Hint: look carefully at the cobweb diagram as a increases through 4.)
3. Consider the system of nonlinear difference equations:

$$x_{n+1} = x_n^2 \tag{1}$$

$$y_{n+1} = x_n + y_n \tag{2}$$

   Find the fixed points and check their stabilities.

4. Consider the system of nonlinear difference equations:

$$x_{n+1} = \mu - \delta y_n - x_n^2 \tag{3}$$

$$y_{n+1} = x_n \tag{4}$$

Find the fixed points and check their stabilities.

5. Consider the following simple model of competition:

$$A_{n+1} = \mu_1 A_n - \mu_3 A_n B_n \tag{5}$$

$$B_{n+1} = \mu_2 B_n - \mu_4 A_n B_n \tag{6}$$

where μ_1, μ_2, μ_3, and μ_4 are positive constants. We call this a *competition* model because the interactions of species A_n and B_n (which we model as the product of the two species) have a negative impact on the evolution of each — notice the minus sign preceding the $A_n B_n$ term of each difference equation.

(a) Find the fixed points.

(b) Determine the stabilities of the fixed points for the specific case of $\mu_1 = 1.2, \mu_2 = 1.3, \mu_3 = 0.001$, and $\mu_4 = 0.002$.