Mathematical Models in the Life Sciences

Assignment 5

Due: 10:00 AM March 5, 2008

You are free to use any books or notes as you work these problems. You are encouraged to work with other members of the class, but not to the point where you simply copy another's work. Also feel free to ask me any questions about these problems. Your work should be neatly and carefully written, and be sure to show all of your work. Solutions to the collected problems will be available following the due date.

- READING: Edelstein-Keshet, 4.0-4.8
- OPTIONAL READING [see course webpage]:
  R. Pearl and L. J. Reed, *On the Rate of Growth of the Population of the United States Since 1790 and its Mathematical Representation*, PNAS, 1920.
- PROBLEMS:

  1. Edelstein-Keshet — Chapter 4: 1, 3, 5.
  2. Consider the nonlinear **difference** equation:
  $$x_{n+1} = \frac{1}{2}x_n - x_n^2 \tag{1}$$
  Find the fixed points and check their stabilities.
  3. Consider the nonlinear **differential** equation:
  $$\dot{x} = \frac{1}{2}x - x^2 \tag{2}$$
  Find the fixed points and check their stabilities. Compare to your results for the previous problem.
  4. For each differential equation below, find the fixed points and check their stabilities.
  $$\dot{x} = \sin x \tag{3}$$
  $$\dot{x} = x - x^3 \tag{4}$$
  $$\dot{x} = x^4 - x^2 \tag{5}$$
  $$\dot{x} = a + x^2, x \text{ real.} \tag{6}$$

5. Consider:

$$\dot{x} = -x^3 \tag{7}$$

Find the fixed point and check its stability — you'll find the stability test at the origin *inconclusive*. Can you use other reasoning to determine the stability of this fixed point?