Mathematical Models in the Life Sciences

Assignment 7

Due: 10:00 AM March 31, 2008

You are free to use any books or notes as you work on these problems. You are encouraged to work with other members of the class, but not to the point where you simply copy another's work. Also feel free to ask me any questions about these problems. Your work should be neatly and carefully written, and be sure to show all of your work. Solutions to the collected problems will be available following the due date.

- Reading: Edelstein-Keshet, 5.2-5.10
- Reminder: Project prospectus due March 26.
- Reminder: Test 2, Friday, April 4.
- Course webpage: http://math.bu.edu/people/mak/MA565
- Problems:
  1. Edelstein-Keshet — Chapter 5: 1b, 1c, 1e; 5a, 5b.
  2. In problems 1b, 1c, and 1e (Chapter 5), you plotted the *direction field* and used it to sketch solutions for each equation. For these same equations, plot $\dot{y}$ versus y and determine the 1-dimensional *phase portrait* of each ODE.
  3. Sketch trajectories in the xy phase plane (see attached page).
  4. Computer software makes drawing the *1-dimensional* direction field much easier. See the course webpage for a link to online (Java) software to plot the 1-dimensional direction field of a function. Use this software to plot the direction field of:
  $$\dot{x} = x - x^3 \tag{1}$$
  When you plot the direction field, make sure to click on the plot window. When you do, you'll select an initial condition and compute forward / backward trajectories from this point. Examine some of these trajectories and determine the fate of the dynamics as time evolves forward. Please print out a plot you create (with the direction field and some trajectories) and hand it in.

5. The same goes for the *2-dimensional* phase plane — computer routines exist to plot the grid of tangent lines. See the course webpage for a link to online (Java) software and plot the 2-dimensional direction field for the system:

$$\dot{x} = x - x^3 - y \tag{2}$$

$$\dot{y} = x \tag{3}$$

When plotting this function consider $-4 < x < 4$ and $-4 < y < 4$. Also, make sure to click on the plot window: when you do, you'll select an initial condition and compute a forward and backward trajectory from that point. Do you find any fixed points? What happens to the dynamics? Please print out a plot you create (with the direction field and some trajectories) and hand it in.