Mathematical Models in the Life Sciences

Assignment 8

Due: 10:00 AM April 14, 2008

You are free to use any books or notes as you work on these problems. You are encouraged to work with other members of the class, but not to the point where you simply copy another's work. Also feel free to ask me any questions about these problems. Your work should be neatly and carefully written, and be sure to show all of your work. Solutions to the collected problems will be available following the due date.

- READING: Edelstein-Keshet, 6.0-6.3, 6.6-6.7
- COURSE WEBPAGE: http://math.bu.edu/people/mak/MA565
- PROBLEMS:

1. Edelstein-Keshet — Chapter 6: 17
2. We claimed in class that oscillations occur in the predator-prey system. Let's check this claim numerically. Start with Equation (7) in Chapter 6 of your textbook (page 219) and set $a = b = c = d = 1$. Then use computer software (such as *PPLANE* — see course webpage) to plot the two-dimensional direction field and some trajectories for the predator-prey system. Do you observe oscillations? Do you observe a fixed point? Hand in your plot and answers to these two questions.
3. We determined in class the constant of motion for the predator-prey system:

$$K = y^a x^c \exp(-by - dx) \tag{1}$$

   Show that K is a constant of motion — namely, show that $\dot{K} = 0$.
4. In the predator-prey model discussed in class, we fixed the prey growth-rate to be constant (parameter a). Let's make the model more realistic and replace a with a density-dependent function. Consider the updated predator-prey model:

$$\dot{x} = r(1 - \frac{x}{\kappa})x - bxy \tag{2}$$

$$\dot{y} = -cy + dxy \tag{3}$$

Note that we've replaced a with the more complicated function $r(1 - x/\kappa)$ and introduced two new parameters (r and κ).

(a) Find the fixed points.

(b) Check their stabilities.

(c) Check your results numerically by plotting the two-dimensional phase plane and some trajectories (you'll have to choose numerical values for the parameters.) Please hand in this plot.

5. Please do problem 32 in Chapter 6 of Edelstein-Keshet, but note that the equations are **incorrect**. Use these equations instead:

$$\dot{x} = rx(1 - \frac{x}{K}) \tag{4}$$

$$\dot{y} = sy(1 - \frac{y}{bx}) \tag{5}$$

In this system of ODEs, the carrying capacity of population y depends on x.

(a) Find the fixed points.

(b) Check their stabilities.

(c) Sketch a phase-plane diagram (by hand or numerically).