# Mathematical Models in the Life Sciences

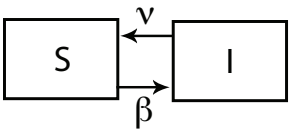
Assignment 9

Due: 10:00 AM April 23, 2008

You are free to use any books or notes as you work on these problems. You are encouraged to work with other members of the class, but not to the point where you simply copy another's work. Also feel free to ask me any questions about these problems. Your work should be neatly and carefully written, and be sure to show all of your work. Solutions to the collected problems will be available following the due date.

- READING: Edelstein-Keshet, 6.6-6.7, 7.0-7.3, 7.5
- PROBLEMS:

1. Edelstein-Keshet: Chapter 6, # 30, **PART (a) ONLY**!
2. In class, we considered a model of two populations in competition. We described in detail the particular case when the two fixed points $(0, K_2)$ and $(K_1, 0)$ were *stable*. We found the fourth (non-trivial) fixed point $(N_1^*, N_2^*)$ was a saddle point, and that the stable approach to this fixed point served as a separatrix. Please consider the case when $(0, K_2)$ and $(K_1, 0)$ are both *unstable* (saddle) fixed points and do the following:
   (a) Sketch the phase plane.
   (b) Determine the stability of the non-trivial fixed point $(N_1^*, N_2^*)$.




3. Consider the SIS model shown in the figure above. In this model, susceptible individuals become sick (at rate $\beta$) through contact with infected individuals. The infected individuals recover (at rate $\nu$) without immunity.
   (a) Write a mathematical model for this system consisting of two coupled ODEs.
   (b) Show that the population size is constant in this model.

(c) Use the constant to eliminate $S$ and reduce the model to a single ODE for $I$.

(d) Solve this single ODE for $\dot{I}$. Do you find a familiar result?