In this lab we'll study the **FitzHugh-Nagumo** model. To do so, we'll use the software package *XPPAUT*.

First, download and install the software from here,

`http://www.math.pitt.edu/~bard/xpp/download.html`

In lab we'll consider the model at two different values of parameter $I = \{0, 0.5\}$. You'll learn to:

- Write an *.ode* file for the FitzHugh-Nagumo model.
- Compute numerical solutions to the model and plot V vs t.
- Plot the phase portrait or scaled direction field.
- Plot the nullclines.
- Find the fixed point(s) numerically and check stability (through examination of eigenvalues).

Questions to answer in lab

1. For $I = 0$:
   (a) How many fixed points exist?
   (b) What is the stability of each?
   (c) Convince yourself of both the number and stability of the fixed point(s) in two or more ways.

TABLE I. **A list of XPPAUT keystroke commands I find useful**

Plot vs time	X-Enter
Erase plot	E
Edit 2-d axes	V-2
Zoom in/out	W-Z or W-O
Zoom to fit	W-F
Run simulation	I-C
Run simulation from last point	I-L
Plot nullcline	N-N
Plot direction field	D-S
Find the fixed points	S-G

Also see

`http://www.math.pitt.edu/~bard/xpp/help/xpphelp.html`

for more commands and useful hints. And

`http://www.math.pitt.edu/~bard/bardware/tut/`

for XPPAUT tutorials.

We used XPPAUT to find numerically the fixed points of the FitzHugh-Nagumo model, and explored how these fixed points changed with parameter I. What if we instead demand an exact formula for the fixed point? To start, fix parameter $I = 0$ and solve the system of equations:

$$0 = \bar{V} - \frac{\bar{V}^3}{3} - \bar{W} \tag{1}$$

$$0 = \phi(\bar{V} + a - b\bar{W}) \tag{2}$$

for $\bar{V}$ and $\bar{W}$. Is this an easy or hard problem? We'll attempt to do this by hand in lab, and see that finding a solution is not so simple. If we *really* want to know this formula, we can use the software package *Mathematica*. This should already be installed on the lab computers, and you can acquire this software and install it for free if you're a BU student. In lab, we'll use Mathematica to solve this system of equations and find an expression for the fixed point $(\bar{V}, \bar{W})$ as a function of I. We'll see it's not pretty . . .

Questions to answer in lab

1. Use Mathematica to find an expression for the fixed point $(\bar{V}, \bar{W})$ as a function of the parameter I. Plot the fixed points as a function of I.

Challenge Problems (Please answer three.)

1. For the FitzHugh-Nagumo model consider the range of parameter values $0 \le I \le 0.5$. At approximatley what parameter value does the model begin spiking? Describe (in words and pictures) how the nullclines change as I increases through this range of parameters. Also describe how the eigenvalues change at the transition from resting to spiking.

2. Consider $I = 0.4$. Plot the nullclines in the (V, W)-plane, and explain how an orbit moves between and along the nullclines. *Suggestion*: Plot the scaled direction field and the periodic trajectory.

3. Using the tools developed in lab, study the following model:

$$\frac{dx}{dt} = f(x) + y + I \tag{3}$$

$$\frac{dy}{dt} = g(x) - y \tag{4}$$

   where $f(x) = -x^3 + ax^2$, and $g(x) = 1 - bx^2$. Fix the parameters $a = 3$ and $b = 5$, and consider the range of values for parameter I: $-0.1 < I < 0.1$. How do the dynamics of the model change as I is varied in this range? To answer this, consider plotting the nullclines and phase portrait, finding the fixed points, and determining their stability for different values of I.

4. Using the tools developed in lab, study the following model:

$$\frac{dV}{dt} = I - g_L(V - E_L) - g_{Na}m_\infty(V)(V - E_{Na}) - g_K n(V - E_K) \tag{5}$$

$$\frac{dn}{dt} = \frac{n_\infty(V) - n}{\tau_n(V)} \tag{6}$$

   where

$$m_\infty(V) = 1/(1 + \exp((-20 - V)/15.0)) \tag{7}$$

$$n_\infty(V) = 1/(1 + \exp((-45 - V)/5)) \tag{8}$$

$$\tau_n(V) = 10 \tag{9}$$

   Fix the parameters $g_L = 8, E_L = -78, g_K = 10, E_K = -90, g_{Na} = 20$ and $E_{Na} = 60$, and consider the range of values for parameter I: $0 < I < 20$. How do the dynamics of the model change as I is varied in this range? To answer this, consider plotting the nullclines and phase portrait, finding the fixed points, and determining their stability for different values of I.

5. Make up your own challenge. Make it interesting.