

MA 294: Applied Abstract Algebra / Spring 2022
Exercise sheet #1

- (1) Show that $n^2 + n$ is even for all $n \in \mathbb{Z}^+$ using induction.
- (2) Show that $n^2 + n$ is even for all $n \in \mathbb{Z}^+$ direction, without using induction.
- (3) Let $S = T = \mathbb{Z}$. Which of the following relations R from S to T is a function $S \rightarrow T$?
 - (a) sRt if $s = t$
 - (b) sRt if $|s - t| < 5$
 - (c) sRt if $s \leq t$
 - (d) sRt if $s^2 = t$
 - (e) sRt if $t^2 = s$
- (4) Exercise 5.2.1. See <https://math.bu.edu/people/medved/Teach/294S2022/Pages/5-42.png>.
- (5) Find a function $\mathbb{Z} \rightarrow \mathbb{Z}$ that is (a) injective but not surjective; (b) surjective but not injective; (c) neither injective nor surjective; (d) both injective and surjective. Same exercise for $\mathbb{Z}^+ \rightarrow \mathbb{Z}^+$.
- (6) Let X, Y be sets and $f : X \rightarrow Y$ a map between them. Show that f is surjective if and only if f is right invertible: that is, if and only if there exists a map $g : Y \rightarrow X$ so that $f \circ g = \text{id}_Y$.
- (7) **A little more challenging:** Exercise 1.22 in Judson's *Abstract Algebra*, a free algebra textbook available online: <http://abstract.ups.edu/aata/exercises-sets.html#mhQ>. Note that A, B , and C are sets.