MA 294: Applied Abstract Algebra / Spring 2022
Homework assignment #2
Due Thursday 2/3/2022 by 4pm

Turn in your work either in class or before 4pm in the envelope hanging on MCS 127.

- Please staple or otherwise connect the pages of your work. There is a stapler in the math department main office.
- Write your name on the front page.
- Consider using a pen rather than a pencil, especially if your pencilwork is smudgy.
- **Challenge problems:** Challenge problems are optional. Please write up your solutions to challenge problems separately. You may also turn in these in later, anytime during the semester.

Integers modulo m: Read sections 13.1 and 13.2.

(1) Exercise 13.1.1

(2) Exercise 13.2.1

Groups: definitions and first examples: Read sections 20.1 and 20.2.

(3) Let Symm($\square$) be the symmetries of a square in the plane. How many elements are there in Symm($\square$)? Explain. Give them names similar to those we gave in class for Symm($\triangle$). Prove that Symm($\square$) is a group under composition.

(4) Give a group table for Symm($\square$) as we gave in class for Symm($\triangle$) or as in Table 20.2.1 (the textbook uses the notation $G_\triangle$ for Symm($\triangle$)).

(5) Exercise 20.2.4

(6) Exercise 20.10.3

(7) (a) Exercise 20.10.6

(b) Now remove one element z from $\mathbb{R}$ so that $\mathbb{R} - \{z\}$ forms a group under the operation $*$ from Exercise 20.10.6. Explain everything. Is this group abelian?

(8) Exercise 20.10.9

Challenge problems

(9) Let S be a set with an associative binary operation $*$ satisfying the following property: There is an element e in G so that

(a) for all a in S we have $a * e = a$

(b) for all a in G there exists an element a' of G so that $a * a' = e$.

Prove that $(S, *)$ is a group and e is its identity element.

(*Hint:* One possible first step is to first show that for any a, the element $b = a' * a$ satisfies $b * b = b$.)

(10) Let S be a set with an associative binary operation $*$ and a (two-sided) identity element e. An element a of S is *left invertible* if there exist some x in S so that $x * a = e$; this x is then a *left inverse* of a. An element $a \in S$ is *right invertible* if there exists y in S so that $a * y = e$; this y is then a *right inverse* of a.

(a) Suppose that $a \in S$ is both left invertible and right invertible. Show that a has a unique left inverse and a unique right inverse, and that these agree.

(b) Show by example that in general left inverses and right inverses need not be unique. (One possibility: let $S = \{\text{functions } \mathbb{Z} \to \mathbb{Z}\}$.... what is $*$? what is e?)