**MA 294: Applied Abstract Algebra / Spring 2022**
**Homework assignment #3**
**Due Thursday 2/10/2022 by 4pm**

Turn in your work either in class or before 4pm in the envelope hanging on MCS 127.

- Please staple or otherwise connect the pages of your work. There is a stapler in the math department main office.
- Write your name on the front page.
- Consider using a pen rather than a pencil, especially if your pencilwork is smudgy.
- **Challenge problems:** Challenge problems are optional. Please write up your solutions to challenge problems separately. You may also turn in these in later, anytime during the semester.

Read section 20.3. In problems (1)–(3), identify which axioms you are using at each step.

(1) Prove the right cancelation law (second part of Theorem 20.3.1).

(2)–(3) Exercises 20.3.2, 20.3.3.
Make sure you can also solve Exercise 20.3.1, but you do not need to write it up.

(4) Let $n$ be a positive integer. If $a$ is an element of a group $G$, we write $a^{-n}$ for $(a^{-1})^n$, the composition of $a^{-1}$ with itself $n$ times. Prove that $a^{-n}$ is the inverse of $a^n$.

Read section 20.5.

(5) Exercise 20.5.2(iii)

(6) Let $G$ and $H$ be two groups with identity elements $1_G$ and $1_H$, respectively.
If $f : G \to H$ is an isomorphism, prove that $f(1_G) = 1_H$.

Given groups $A$, $B$, we can define a natural group structure on the Cartesian product $A \times B$. Read the first full paragraph on p. 269 about this construction.

(7) Exercise 20.6.3.

(8) (a) Give the group table for $\mathbb{Z}_2 \times \mathbb{Z}_2$. (Here $\mathbb{Z}_2$ is the group of order 2 under addition.)
(b) By Exercise 20.5.2(iii), the group $\mathbb{Z}_2 \times \mathbb{Z}_2$ has to be the same as one of the two groups of order 4 that we've studied in class. Which one is it? Explain!

**Challenge questions**

(9) Find all groups of order 5 up to relabeling.
(Can you do the same for groups of order 6? It's a doozy!)

(10) Suppose $G$ is a set with an associative binary operation $*$ so that the "composition table" for $*$ is a latin square. Is $(G, *)$ necessarily a group? Either prove that it is or find a counterexample!