

**MA 294: Applied Abstract Algebra / Spring 2022**  
**Homework assignment #7**  
**Due Thursday 3/31/2022 by 4pm**

Turn in your work either in class or before 4pm in the envelope hanging on MCS 127.

- Please staple or otherwise connect the pages of your work. There is a stapler in the math department main office. Yes, this requires a tiny bit of planning — but I know we can do this! Our grader is anxious that homework set pages might get lost.
- Write your name on the front page. Plan ahead: perhaps you need to carry around a pen for this purpose on Thursdays?
- Consider using a pen rather than a pencil, especially if your pencilwork is smudgy.

Read sections 12.5–12.6, section 20.8 from the middle of p. 275 on, and section 21.1.

- (1) Exercise 12.5.1
- (2) Exercise 12.6.1
- (3) Exercise 12.6.2
- (4) Show that for  $\sigma \in S_n$  we have  $\text{sgn}(\sigma) = (-1)^{n-c(\sigma)}$ .
- (5) Exercise 21.1.1
- (6) Exercise 21.1.2
- (7) Exercise 21.1.3
- (8) Exercise 21.1.4

The following may be a useful observation: Biggs shows that if  $\sigma \in S_n$  be any permutation and  $\tau \in S_n$  is a transposition, then

$$c(\tau\sigma) = c(\sigma) \pm 1$$

(see pp. 137–138 in section 12.6; here  $c(\sigma)$  is the number of cycles in  $\sigma$ , including singletons).

**Challenge question**

- (9) (a) Let  $\sigma \in S_n$  be a  $k$ -cycle. In class we saw that  $\sigma$  can be expressed as a product of  $k - 1$  transpositions. Show that there is no expression for  $\sigma$  as a product of fewer than  $k - 1$  transpositions.
- (b) More generally, can you give a formula for the least number of transpositions that you need to express  $\sigma \in S_n$  as a product of transpositions?