MA 294: Applied Abstract Algebra / Spring 2022
Homework assignment #9
Due Thursday 4/14/2022 by 4pm

Ok to turn your writeup following week, but don't wait to work out the problems!
Edit 14 April 2022 in (4).

Turn in your work either in class or before 4pm in the envelope hanging on MCS 127.

- Please staple or otherwise connect the pages of your work. There is a stapler in the math department main office. Yes, this requires a tiny bit of planning — but I know we can do this! Our grader is anxious that homework set pages might get lost.
- Write your name on the front page. Plan ahead: perhaps you need to carry around a pen for this purpose on Thursdays?
- Consider using a pen rather than a pencil, especially if your pencilwork is smudgy.

Read section 21.4.

(1) Exercise 21.4.1. Make sure to do the enumeration of the necklaces both by hand (that's the sketching them all part) and by using Burnside's orbit-counting lemma (Theorem 21.4).

(2) Exercise 21.4.2

(3) Exercise 21.4.4

(4) ~~Exercise 21.7.3~~ Exercise 21.7.8.
Since this typo was corrected only on the morning of the due date, if you've already solved and written up Exercise 21.7.3, you do not need to redo the problem to turn in. But you should figure out what's going on with Exercise 21.7.8, which is about coloring vertices of a tetrahedron, eventually.

(5) Let G be the group of rotational symmetries of the cube. It may help to have a cube handy when thinking about this problem (ask if you're having trouble finding one).

In (7) on HW #8 we saw that we can view G as a subgroup of S_4 by viewing G as a group of permutations of the diagonals of the cube. For each of the following subsets of elements of G, lists the elements in cycle notation in S_4.

- the stabilizers of vertices (note that each vertex stabilizer element also stabilizes the opposite vertex)
- the stabilizers of faces (note that each face stabilizer element also stabilizes the opposite face)
- the stabilizers of edges (note that each edge stabilizer element also stabilizes the opposite edge)

Show that G is the union of all of these elements. Prove that $G \cong S_4$.