MA 541: Modern Algebra I / Fall 2019
Challenge problem for homework assignment # 1

(1) Let G be a set with an associative binary operation $*$ satisfying the following property: There is an element $\tilde{e}$ in G so that
  (a) for all g in G we have $g * \tilde{e} = g$; and
  (b) for all g in G there exists an element $\tilde{g}'$ of G so that $g\tilde{g}' = \tilde{e}$.
  Prove that $(G, *)$ is a group and $\tilde{e}$ is its identity element.

  Hint: Show that the element $h = \tilde{g}' * g$ satisfies $h * h = h$.