MA 541: Modern Algebra I / Fall 2019
Homework assignment #7
Due 11/5/2019

(0) Read in F...
- ...section 8, the part called "Cayley's Theorem";
- ...section 9, the parts called "Even and Odd Permutations" and "The Alternating Groups";
- ...section 10.

(1) F exercise 9.23

(2) F exercise 9.29

(3) Use the proof of Cayley's Theorem (F Theorem 8.16) to realize the quaternion group Q_8 as subgroup of S_8, as we did in class for $\mathbb{Z}_4$ and D_3.
(That is, list the elements of Q_8 and label them with indices $1 \ldots 8$. For each element of Q_8, give the permutation in S_8 that corresponds to left translation by that element.)

(4) Recall that $D_4 = \langle r, f : r^4 = f^2 = rfrf = 1 \rangle$.
  (a) Find the left cosets of the subgroup $\langle f \rangle$ in D_4. Find the rights cosets of $\langle f \rangle$ in D_4. Are the two sets of cosets the same?
  (b) Same question for the subgroup $\langle r^2 \rangle$ of D_4.

(5) **Right cosets:** Let G be a group, and $H \leq G$ a subgroup. Recall that for any $g \in G$, the set Hg is a *right coset* of H in G.
  (a) Show that two right cosets of H in G either coincide or are disjoint.
  (b) Show that the relation $\sim_R$ on G given by $g \sim_R h$ if and only if $gh^{-1} \in H$ is an equivalence relation.
  (c) Show that the equivalence class of $g \in G$ under the equivalence relation from (b) is the right coset Hg.

(6) Let G be a group. For $a, b \in G$, we say that a is *conjugate* to b (or a is a *conjugate* of b) if there exists some $g \in G$ such that $a = gbg^{-1}$. The element a then is the result of *conjugating* b *by* g.
[Compare to problem 4(a) on the midterm: you essentially showed that for groups conjugation is a self-similarity (better known as an *automorphism*).]
  (a) Prove that if a is conjugate to b, then there exists some $h \in G$ such that $a = h^{-1}bh$.
  (b) Find the result of conjugating the element $(1\ 7\ 5\ 2\ 3)(4\ 6)$ of S_7 by $(2\ 4)$. Do you notice anything?
  (c) Find all the elements of S_3 that are conjugate to $(1\ 2)$.
  (d) Find all the elements of A_4 that are conjugate to $(1\ 2\ 3)$.