# MA 542: Modern Algebra II / Spring 2023
## Homework assignment #2
## Due ~~Friday 2/10/23~~ Monday 2/13/23 in class

Final version [Edit 12 Feb 23: minor typo in (13). Also hint added.]

Make sure to read BB through section 4.3.

Exercises from BB:

(1) 4.1.2

(2) 4.1.5(b)(d)

(3) 4.1.6

(4) 4.1.7
(*Suggestion:* Prove that if $r$ and $s$ are orders of elements in an abelian group, then so is $\mathrm{lcm}[r, s]$. If $m$ is the maximal order of any element of $\mathbb{Z}_p^\times$, what can you say about $x^m - 1$?)

(5) 4.1.16. Have you seen this field before?

(6) 4.2.2(a,b,c)

(7) 4.2.5(d) and 4.2.7(d)

Additional exercises:

(8) Show that $x^2 + x + 1$ is irreducible in $\mathbb{R}[x]$, so that $F := \mathbb{R}[x]/(x^2 + x + 1)$ is a field. Is $F$ isomorphic to $\mathbb{C}$? Either find an isomorphism or show that none exists.

(9) List all irreducible polynomials of degree 2 in $\mathbb{Z}_3$. Construct a finite field with 9 elements.

(10) Construct a finite field with 16 elements.

(11) Let $F := \mathbb{Z}_5[x]/(x^2 + 2)$. Show that $F$ is a field. Let $\alpha$ be the image of $x$ in $F$.
What is the order of $\alpha$ in $F^\times$?

(12) Let $F$ be a finite field of size $q$. Show that $\alpha^q = \alpha$ for all $\alpha \in F$.

(13) Prove that any finite subgroup of $\mathbb{C}^\times$ is cyclic.
(*Hint:* Can you use the same methods as for (4)?)

(14)
 (a) Factor $y^3 - y$ in $\mathbb{Z}_3[y]$ into irreducibles.
 (b) Factor $y^5 - y$ in $\mathbb{Z}_5[y]$ into irreducibles.
 (c) Factor $y^4 - y$ in $F[y]$, where $F = \mathbb{Z}_2[x]/(x^2 + x + 1)$ is a field of order 4, into irreducibles.