NAME: _____

MA 542: Modern Algebra II / Spring 2023
Quiz / 17 February 2023

(1) Let $f(x) = x^3 + 8x^2 - 4x + 6$.
For each statement below, explain whether it is true or false, and why.

(a) Viewed in $\mathbb{Q}[x]$, the polynomial $f(x)$ is irreducible.

(b) Viewed in $\mathbb{R}[x]$, the polynomial $f(x)$ is irreducible.

(c) Viewed in $\mathbb{C}[x]$, the polynomial $f(x)$ is irreducible.

(d) Viewed in $\mathbb{Z}_2[x]$, the polynomial $f(x)$ is irreducible.

(e) Viewed in $\mathbb{Z}_5[x]$, the polynomial $f(x)$ is irreducible.

(2) Let $F := \mathbb{Z}_3[x]/(x^3 + 2x + 1)$.

(a) Explain why F is a field.

(b) How many elements does F have? Explain.

(c) What is the inverse of $[x]$ in F? Explain.

(d) Find a root of the polynomial $y^3 + 2y + 1$ in $F[y]$.

(e) **Optional:** Can you find *another* root of $y^3 + 2y + 1$ in $F[y]$?